Regulating Liquidity Risk in Mutual Funds

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Abstract

I analyze the effects of liquidity risk regulation in a model of investors, mutual funds, and the underlying asset market. Investor redemptions lead mutual funds to sell assets, which may result in fire sales if market liquidity, driven by the anticipation of fire sales, is scarce. Mutual funds optimally choose to pass fire sales of their assets on to investors. Pecuniary externalities make liquidity supply to the underlying asset market inefficiently low. Regulatory policies, liquidity requirements for mutual funds, and redemption gates have adverse effects on liquidity provision to the asset market and may increase the incidence of fire sales.

Keywords: Mutual funds, fire sales, financial regulation, market liquidity

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Introduction

Globally, more than USD30 trillion in assets are held in open-ended mutual funds that offer short-term redemptions while investing in longer-dated and potentially illiquid assets such as corporate bonds. After the COVID-19 pandemic put severe pressure on the industry, the Federal Reserve called for structural reforms based on the assessment that "fixed-income mutual funds continue to be vulnerable to large, sudden redemptions, and sizable outflows can still lead to a deterioration in market liquidity of underlying assets" (Federal Reserve Board, 2020). This quote reflects an ongoing debate between academics and policymakers about the financial stability implications of illiquid mutual funds, their impact on asset market liquidity and the effects of regulatory interventions.

To address these questions I develop a tractable model of investor redemptions in mutual funds where market liquidity and fire sales are endogenously determined. Liquidity providers (for example hedge funds) build up arbitrage capital with the objective of purchasing under-valued assets from mutual funds in fire sales, thereby stabilizing market liquidity and asset prices. However, a pecuniary externality affects liquidity supply and results in a (constrained) inefficient competitive equilibrium with an excessive likelihood of fire sales. I study the effects of regulatory policy proposals aiming to mitigate liquidity risk in mutual funds, namely liquidity requirements and redemption gates. The analysis reveals important and rarely discussed general equilibrium effects of regulatory policies on market liquidity: reducing the need for asset (fire) sales by mutual funds lowers liquidity providers' incentives to build up arbitrage capital. This adverse effect on liquidity supply to asset markets may be strong enough to outweigh the benefits of regulation and result in an equilibrium with less market liquidity and more fire sales.

The analysis is based on a three-date equilibrium model of the mutual fund industry with three types of risk-neutral agents: investors, mutual funds (MFs), and liquidity providers. At the initial date investors invest their endowment in a MF in exchange for MF shares. MFs act as financial intermediaries and invest the collected funds in a risky long-term asset, for example a portfolio of corporate bonds. The role of MFs as intermediaries can be rationalized by the fact that many asset classes such as corporate bonds are not readily available to retail investors. Liquidity providers invest their endowment in a portfolio of a long-term illiquid investment project and a short-term liquid asset. Liquid funds can subsequently be used to purchase some of the MF's risky asset at the interim date.

The risky asset's success probability is determined by the realization of an aggregate shock at the start of the interim date. After observing the risky asset's success probability ("quality"), investors gain access to a short-term investment project at the interim date. To invest in this project, investors can redeem some of their MF shares at the interim date, thereby raising funds to invest in the short-term project, or keep their shares until the final date. The share price at both dates is determined by a contract between the MF and investors which specifies how the share price reacts to the aggregate state of the economy. Perfect competition in the MF industry results in contracts designed to maximize investors' expected payoff. MFs accommodate redemptions at the interim date by selling some of the risky asset to liquidity providers in a competitive asset market.

I characterize the competitive equilibrium of the risky asset market, redemptions and share prices as well as liquidity providers' ex-ante portfolio choice. In equilibrium, the risky asset may trade at its fundamental price or at a fire sale discount. Which case obtains depends on the risky asset's quality and the available market liquidity, that is, the aggregate amount of liquid assets in the hands of liquidity providers. In particular, a decrease in market liquidity increases fire-sale discounts as liquidity providers may lack the funds to purchase the risky asset at its fundamental value, leading to cash-in-the-market pricing as in Allen and Gale (1994). In contrast, a decrease in asset quality leads investors to sell more of their risky asset holdings. MFs optimally choose to pass changes in the market value of their assets fully on to investors, that is, shares are marked to market.

Liquidity providers' portfolio choice at the initial date endogenously determines asset market liquidity and is key for the analysis of (in)efficiencies in the model. While the long-term investment project is attractive due to its safe payoff at the final date, it is illiquid and does not generate funds for risky asset purchases at the interim date. The illiquidity of the long-term project, coupled with the potential of purchasing the (undervalued) risky asset at fire-sale prices, induces liquidity providers to hold the liquid asset. Interestingly, I show that the equilibrium of the model is constrained inefficient, in the sense that a social planner choosing the initial investment in the liquid asset could improve upon the equilibrium allocation. In particular, the social planner would increase liquidity provision, thereby reducing the likelihood of fire sales in the competitive asset market.

The underprovision of liquidity in equilibrium arises due to a pecuniary externality that is driven by the assumption of market incompleteness: Liquidity providers can finance risky asset purchases from MFs at the interim date only with their liquid asset holdings. Therefore, MFs' revenue from selling the risky asset to pay redeeming investors is constrained by the available liquidity in the market. Liquidity providers fail to internalize the impact of their liquidity holdings on the equilibrium price of the risky asset and ultimately on redeeming investors' investment in the productive short-term project. In other words liquidity providers do not fully account for the social value of holding the liquid asset at the initial date, which leads to inefficiently low market liquidity.

I analyze two policy proposals aiming to mitigate liquidity risk in MFs: first, a liquidity requirement that leads MFs to hold liquid assets in their portfolio, and second, a redemption gate that restricts investor redemptions during times of market turmoil. The direct effect, for a given level of asset market liquidity, of both policies is to reduce the need for asset (fire) sales by MFs, thereby lowering the likelihood of fire sales. In equilibrium, however, liquidity supply to the asset market decreases as the policies lower the expected returns to holding liquidity. The net effect of both policies is determined by the relative strength of their direct and indirect effects. Since redemption gates insulate MFs from the most severe fire sales, when returns to liquidity providers' arbitrage capital are highest, they result in an equilibrium with lower market liquidity and more fire sales. By contrast, liquidity requirements leave sufficient incentives to build up arbitrage capital such that market liquidity increases, leading to a reduction in the likelihood of fire sales.

The positive net effect of liquidity requirements raises the question whether MFs would build up efficient liquidity buffers in the absence of a regulatory policy mandate. I study this question in an extension of the model in which MFs can invest in a portfolio of the risky asset and the liquid asset at the initial date. Mutual fund liquidity reduces the need for asset (fire) sales at the interim date but comes at the cost of foregone returns from investing more in the risky long-term asset. Interestingly the results show that when markets are incomplete, MFs hold inefficiently low levels of liquidity buffers compared to a social planner choosing the liquid asset holdings by both liquidity providers and MFs. This result further strengthens the case for liquidity requirements for MFs.

Related literature. This paper is related to a rich literature analyzing liquidity provision in financial markets and the link with asset prices. Following Allen and Gale (2004, 2005), several papers have analyzed how cash-in-the-market pricing affects financial institutions' liquidity decisions and the possibility of central bank or public intervention.[1] Acharya, Shin, and Yorulmazer (2013); Gale and Yorulmazer (2013); and Gorton and Huang (2004) focus on the strategic motive for holding liquid assets, where liquidity allows to capitalize on profitable opportunities such as purchasing undervalued assets in fire sales. Acharya, Shin, and Yorulmazer (2011) show that the expectation of fire sales may lead to excessive liquidity hoarding from a welfare perspective and focus on the interaction between regulatory interventions and ex-ante liquidity choices.

Arbitrageurs wanting to profit from fire sales may face financing frictions due to principal-agent problems. The resulting "limits of arbitrage" can entrench fire-sale prices (Shleifer and Vishny, 1997; and Mitchell, Pedersen, and Pulvino, 2007). Bolton, Santos, and Scheinkman (2011) study the interplay between endogenous liquidity choices and private information.[2] Dávila and Korinek (2017) analyze pecuniary externalities, which generate inefficiencies in models with cash-in-the-market pricing.

The paper is also related to the literature studying the potential of non-banking institutions to destabilize financial markets. Mutual funds investing in illiquid securities may be vulnerable to simultaneous investor outflows because the liquidity mismatch between the funds' investments and the liquidity offered to its investors leads to predictable declines in share prices following redemptions, thereby generating a first-mover advantage (Chen, Goldstein, and Jiang, 2010; Goldstein, Jiang, and Ng, 2017; and Jin et al., 2020). This mechanism has figured prominently in the policy

1 Papers in this tradition include Allen, Carletti, and Gale (2009); Freixas, Martin, and Skeie (2011); and Gale and Yorulmazer (2013).

2 Other papers in this tradition include Malherbe (2014); Ahn et al. (2018); and Heider, Hoerova, and Holthausen (2015).

debates (ECB, 2019; and BoE, 2019). Falato, Goldstein, and Hortaçsu (2020) document that both the illiquidity of fund assets and funds' vulnerability to fire sales were important factors in determining investor redemptions during the Covid crisis. Jiang et al. (2020) show empirically that mutual funds' liquidity transformation can lead to fragility in underlying asset markets.

Lastly, the paper relates to the literature on mutual funds' management of redemption risk. Morris, Shim, and Shin (2017) focus on liquidity management by asset managers and argue that funds sell more assets than required to cover outflows, suggesting a cash hoarding channel. Chernenko and Sunderam (2020) find that mutual funds' liquidity buffers are insufficient to eliminate fire sales. Zeng (2017) argues that funds' liquidity buffers may exacerbate investor runs because re-building cash buffers requires predictable sales of illiquid assets, which trigger declines in share prices. Li et al. (2020) find evidence that redemption gates and liquidity fees may have exacerbated the run on prime money market funds during the COVID-19 crisis. Cutura, Parise, and Schrimpf (2020) document that underperforming corporate bond funds tilt their portfolios toward more liquid, lower-yield securities, which successfully mitigates investor outflows.

Structure of the paper. Section 2 presents the model of investor redemptions in mutual funds with endogenous fire sales and liquidity provision. Section 3 studies the equilibrium in a laissez-faire economy. Section 4 presents the first and second-best benchmarks. The effects of regulatory interventions in the mutual fund industry are discussed in Section 5 before Section 6 concludes. Appendix 7 highlights the core modeling assumptions generating the pecuniary externality. Appendices 8 and 9, respectively, analyze the equilibrium with liquidity requirements and endogenous mutual fund liquidity buffers. Finally, Appendix 10 derives the equilibrium under redemption gates.

Model

This section develops a model of investor redemptions in mutual funds with endogenous asset market liquidity and fire sales. Since many asset classes are not easily accessible to retail investors, mutual funds act as financial intermediaries and invest in risky assets on behalf of their investors. Investors, in turn, hold mutual fund shares which can be redeemed early or be held until maturity at their current share price. A contract signed between investors and a mutual fund at the initial date determines how the share price varies with the market value of the assets in the fund's portfolio (which depends on the aggregate state of the economy). Perfect competition in the mutual fund industry results in contracts designed to maximize investors' expected consumption.

Setting. Consider an economy with three dates indexed by $t \in \{0,1,2\}$. There is a single, homogeneous consumption good which serves as the numéraire. There are three classes of risk-neutral agents in the economy: investors, liquidity providers, and mutual funds. There are two divisible financial assets in the economy: a safe, liquid asset with a unit return and a risky, long-term asset with payoff at date 2 given by

$$\tilde{y} = \begin{cases} 0 & \text{with probability } 1-\pi \\ y & \text{with probability } \pi \end{cases}$$

where $y > 0$. The success probability π is an aggregate asset quality shock that is observed by all agents at date 1. From the perspective of date 0, the shock has a cumulative distribution function $H(\pi)$.

Investors. There is a continuum of investors with measure one in each mutual fund. Investors consume only at the final date and do not discount their future consumption. At date 0, each investor exchanges her unit endowment of the consumption good for a divisible unit amount of mutual fund shares, which can be redeemed at date 1 and 2. At date 1, each investor gains access to a short-term investment project that transforms k units of the consumption good into $f(k)$ units of the consumption good at date 2. The production function is increasing and concave and satisfies $f'(0)=\infty$ and $kf''(k)+f'(k)>0$.

Investors raise funds to invest in the short-term project by redeeming some of their mutual fund shares. Let s_t denote the price of a share in the mutual fund at date $t = 1,2$, which will be a function of the price p of the risky asset at date 1. Redeeming x mutual fund shares at the given share price s_1 yields xs_1 units of the consumption good at date 1 to invest in the short-term project. At the end of date 1, each investor is left with $(1-x)$ mutual fund shares with an expected share price of $s_2^e \equiv \mathbb{E}[s_2]$.

Contingent on the realization of the asset quality shock π, the representative investor chooses the number of shares to redeem at date 1 to maximize her expected consumption at date 2, that is

$$\max_{x\in[0,1]} \left\{f\left(xs_1\right)+\left(1-x\right)s_2^e\right\}.$$

The first-order condition that characterizes an interior solution to this problem equates the marginal expected return of the mutual fund shares with the marginal return from investing the proceeds from redemptions in the short-term project:

Equation 1

$$s_1 f'\left(xs_1\right)=s_2^e.$$

Whenever there is an interior solution, differentiating the first-order condition in Equation 1 yields

$$\frac{dx}{ds_1}=-\frac{xs_1 f''\left(xs_1\right)+f'\left(xs_1\right)}{s_1^2 f''\left(xs_1\right)}>0,$$

due to the assumptions $f''(k)<0$ and $kf''(k)+f'(k)>0$. However, when the share price s_1 is sufficiently high, we may have a corner solution in which the investor redeems all of her mutual fund shares. This will be the case whenever $s_1 f'\left(xs_1\right)\geq s_2^e$. Let $x\left(s_1\right)$ denote the solution to the investor's problem.

I can illustrate these results with a simple parametric example, which will be used in the numerical analysis below. In particular, suppose that $f(k)=2\sqrt{k}$. Then it is immediate to show that

$$x\left(s_1\right)=\min\left\{s_1\left(s_2^e\right)^{-2},1\right\}.$$

Liquidity providers. There is a continuum of liquidity providers with measure one. At date 0, each liquidity provider is endowed with w units of the consumption good and has access to two investment opportunities: first, a long-term investment project that transforms k units of the consumption good at date 0 into $g(k)$ units of the consumption good at date 2. The production function is increasing and concave and satisfies $g'(0)=\infty$. Second, they can invest in a short-term liquid asset with a safe gross return of one between dates 0 and 1 and between dates 1 and 2. At date 0, each liquidity provider chooses a portfolio consisting of m units of the liquid asset and an investment of $w-m$ in the long-term project. At date 1, after the realization of the asset quality shock, liquidity providers can invest up to m units of the liquid asset to purchase q^D units of the mutual fund's risky asset with an expected value of πy at the unit price p.

At date 1, the representative liquidity provider chooses her demand for the risky asset to maximize her consumption at date 2, that is,

$$\max_{q^D>0} \left\{g(w-m)+m+q^D\left[\pi y-p\right]\right\}$$

subject to the resource constraint

$$q^D \leq \frac{m}{p}.$$

The resource constraint states that risky asset purchases can be financed only with the liquid asset in the liquidity provider's portfolio. This restriction is akin to a liquid assets-in-advance constraint as highlighted by Gorton and Huang (2004) and crucial to generating some of the inefficiencies, which will be discussed later on.

Since the objective function is linear in q^D, it is immediate to describe the optimal demand. When the risky asset trades at its expected (fundamental) value, liquidity providers generate no surplus by purchasing it and are thus indifferent between holding on to their liquid asset or trading with mutual funds. Instead, when the risky asset trades at a fire-sale discount, purchasing it becomes profitable and liquidity providers completely exhaust their liqu id reserves. Let $q^D(p;\pi,m)$ denote the solution to the liquidity provider's problem at date 1, where

Equation 2

$$q^D(p;\pi,m) = \begin{cases} 0 & \text{if } p > \pi y \\ \in \left[0, \frac{m}{p}\right] & \text{if } p = \pi y \\ \frac{m}{p} & \text{if } p < \pi y \end{cases}.$$

Anticipating their demand for the risky asset and the resulting need for liquidity at date 1, liquidity providers choose their portfolio at date 0. The portfolio is designed to maximize their expected consumption at date 2, which consists of the long-term project's payoff and the return of holding the liquid asset, which may be used to purchase some of the risky asset at the interim date or be kept until the final date.

The representative liquidity provider chooses her investment in the liquid asset to maximize her consumption at date 2, that is,

$$\max_{m \in [0,w]} \left\{ g(w-m) + m + \mathbb{E}\left[q^D(p;\pi,m)(\pi y - p) \right] \right\}$$

where $q^D(p;\pi,m)$ denotes her demand for the risky asset at date 1 described by Equation 2. Note that, in equilibrium, the price of the risky asset p varies with the realization of the asset quality shock π and is therefore inside the expectation operator.

Mutual funds. There is a continuum of mutual funds with unit mass. At date 0, each mutual fund collects investors' unit endowment and invests it on their behalf in one unit of the divisible risky long-term asset with payoff at date 2 given by $\tilde{y}$.

The problem of each mutual fund consists in designing a contract for its investors at date 0 that maximizes their expected consumption at date 2. The contract specifies how the interim share price s_1 reacts to the market value of the fund's portfolio at date 1. Since the risky asset does not generate any payoff at date 1, mutual funds sell q^S units of the risky asset at the market price p in order to accommodate investors' redemptions.

The representative mutual fund's contract is described by a pair $\left(q^S, s_1\right)$ chosen so as to maximize investors' expected consumption:

$$\max_{s_1(\pi), q^S(\pi) \in [0,1]} \quad \left\{\mathbb{E}\left[f\left(x\left(s_1\right)s_1\right) + \left(1 - x\left(s_1\right)\right)s_2\right]\right\},$$

subject to investors' optimal redemption decision in Equation 1, the mutual fund's budget constraint

Equation 3

$$x\left(s_1\right)s_1 = q^S p,$$

and the value of outstanding shares at date 2

Equation 4

$$s_2 = \frac{\left(1 - q^S\right)\tilde{y}}{1 - x\left(s_1\right)}.$$

The budget constraint in Equation 3 ensures that risky asset sales are sufficient to accommodate redemptions. The value of a share at date 2 represents the value of the mutual fund's remaining units of the risky asset relative to the number of outstanding shares.

The model with mutual funds FIGURE 1

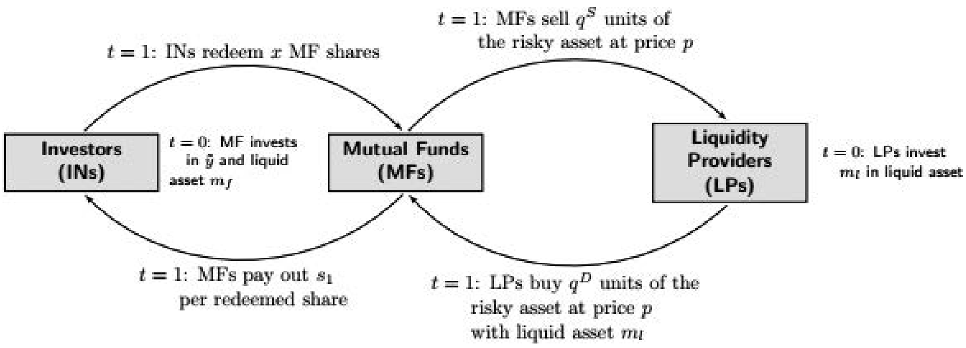


Figure 1 illustrates the timing of the model. At date 0 mutual funds design the contract and liquidity providers choose their liquid asset holdings. At date 1, after the realization of the aggregate shock π, investors gain access to the short-term project f, leading them to redeem some mutual fund shares at the contractually specified share price s_1. Mutual funds accommodate these redemptions by selling some of their risky asset holdings at the market price p. Liquidity providers determine how many units of the risky asset to purchase at the price p given the liquid funds in their portfolio.

Note that the mutual fund contract is designed at date 0, but the interim share price and the mutual fund's supply of the risky asset at date 1 do not depend on any actions by the mutual fund or investors at date 0 other than the contract terms set at date 0. The optimal contracting problem can therefore be solved independently for each possible aggregate state of the economy at date 1, that is, it reduces to a pointwise optimization problem. The interim share price and the supply of the risky asset are thus functions of the aggregate shock π. Moreover, the mutual fund's budget constraint in Equation 3 highlights that the fund's choices depend on the market value of its risky asset holdings *p*. Hence, the objective function of the representative mutual fund's problem can be written as

Equation 5

$$\max_{s_1(p;\pi),q^S(p;\pi)\in[0,1]} \left\{ f\left(x\left(s_1(p;\pi)\right)s_1(p;\pi)\right) + \left(1-q^S(p;\pi)\right)\pi y \right\}$$

where I have substituted the value of non-redeemed shares s_2 from Equation 4 to highlight that it is fully determined by choosing $s_1(p;\pi)$ and $q^S(p;\pi)$. Similarly, replacing s_2^e in investors' first-order condition determining redemptions in Equation 1 and re-arranging yields

Equation 6

$$s_1(p;\pi)\left[1-x\left(s_1(p;\pi)\right)\right] = \frac{\left(1-q^S(p;\pi)\right)\pi y}{f'\left(x\left(s_1(p;\pi)\right)s_1(p;\pi)\right)}.$$

To solve the mutual fund's contracting problem, I proceed by utilizing the budget constraint in Equation 3 to replace $x\left(s_1(p;\pi)\right) = q^S(p;\pi)p / s_1(p;\pi)$ in the objective function Equation 5 and investors' modified first-order condition in Equation 6, which yields the simplified problem:

Equation 7

$$\max_{s_1(p;\pi),q^S(p;\pi)\in[0,1]} \left\{ f\left(q^S(p;\pi)p\right) + \left(1-q^S(p;\pi)\right)\pi y \right\}$$

$$\text{subject to} \quad s_1(p;\pi) = \frac{\left(1-q^S(p;\pi)\right)\pi y}{f'\left(q^S(p;\pi)p\right)} + q^S(p;\pi)p.$$

The constraint determines the interim share price as a function of the risky asset's market price and the extent of the fund's asset sales. Therefore I can proceed by initially ignoring this constraint to determine the optimal supply of the risky asset $q^S(p;\pi)$, which is sufficient to find the equilibrium price of the risky asset *p*. Then I utilize the pair $\left(q^S(p;\pi),p\right)$ to recover the optimal share price $s_1(p;\pi)$ implied by the constraint in Equation 7.

By initially ignoring the constraint in Equation 7, the representative mutual fund's problem reduces to determining the optimal supply of the risky asset at date 1, conditional on the realized aggregate shock π:

$$\max_{q^S(p;\pi)\in[0,1]} \left\{ f\left(q^S(p;\pi)p\right) + \left(1-q^S(p;\pi)\right)\pi y \right\},$$

The first-order condition that characterizes an interior solution to this problem equates the marginal expected return of the long-term asset with the marginal return from investing the liquid funds raised through asset sales in the short-term project, that is

$$pf'\left(pq^S\left(p;\pi\right)\right)=\pi y.$$

Whenever there is an interior solution, I can differentiate the first-order condition and show that $dq^S\left(p;\pi\right)/ds_1\left(p;\pi\right)>0$ using the assumptions on the production function *f*. However, when the price *p* of the risky asset is sufficiently high, there may be a corner solution in which the mutual fund sells the entire holding of the risky asset. This will be the case whenever $pf'\left(p\right)\geq\pi y$. Let $\hat{p}$ denote the price at which this holds with equality such that the mutual fund sells all of its risky asset holdings if the price satisfies $p\geq\hat{p}$. Let $q^S\left(p;\pi\right)$ denote the solution to the fund's problem. In the simple parametric example with $f\left(k\right)=2\sqrt{k}$, I obtain $q^S\left(p;\pi\right)=\min\left\{p\left(\pi y\right)^{-2},1\right\}$ and $\hat{p}=\left(\pi y\right)^2$.

Competitive equilibrium

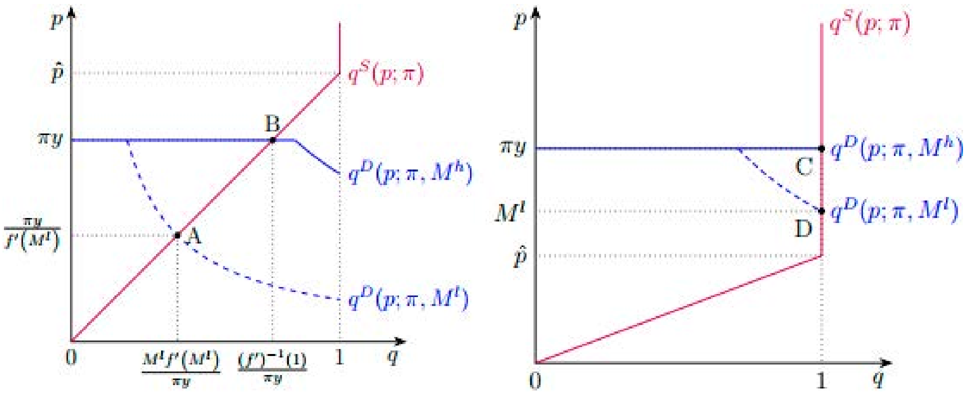
This section derives the competitive equilibrium of the model, including the optimal contract between investors and mutual funds, and the market for the risky asset. The equilibrium analysis proceeds in a backward fashion: first, I derive the equilibrium on the asset market and the mutual fund contract at date 1 before analyzing liquidity providers' investment in the liquid asset at date 0.

Asset market clearing

The equilibrium on the market for the risky asset at date 1 is determined by the intersection of mutual funds' supply and liquidity providers' demand for the risky asset. Trading on the risky asset market takes place after all agents observe the realization π of the asset quality shock. Each liquidity provider has a predetermined amount of the liquid asset m in their portfolio which can be used for risky asset purchases. Let M denote the *aggregate* amount of the liquid asset in the hands of liquidity providers, which will subsequently be referred to as market liquidity. This section highlights that the realized asset quality and market liquidity jointly determine the equilibrium in the market for risky assets.

Liquidity providers' demand curve in Equation 2 has a horizontal and a downward-sloping part: they are indifferent between purchasing the asset or not if it trades at its fundamental price but purchase as much of it as they can afford at fire-sale discounts. Mutual funds' supply curve in Equation 1 consists of an upward-sloping and a vertical part, since a sufficiently high price leads them to sell all of their risky asset holdings. With supply and demand consisting of two parts each, there are four possible intersections. Figure 2 depicts supply and demand for the risky asset and illustrates the candidate equilibria. I now analyze each intersection and derive the conditions under which the respective equilibrium obtains.

Demand and supply for the risky asset at date 1 FIGURE 2



(a) Case 1: $\hat{p} > \pi y$ (b) Case 2: $\hat{p} < \pi y$

Note: This figure depicts mutual funds' supply of the risky asset, q^s, and liquidity providers' demand, q^D, for different levels of market liquidity $\$M^s$, $s=\{h,l\}$ with $M^h > M^l$. Panel a depicts a case where the lowest price at which mutual funds are willing to sell all of the risky asset is above the price at which liquidity providers would buy it ($\hat{p} > \pi y$). In contrast, Panel b depicts the case in which funds may sell all of their risky asset holdings.

First, suppose that the equilibrium is located on the upward-sloping part of the supply curve and the decreasing part of the demand curve (Point A in Figure 2(a)). The downward-sloping demand curve implies that liquidity providers' resource constraint binds $(pq = M)$, which can be plugged into mutual funds' first-order condition in Equation 1 to obtain the market clearing price and quantity of the asset:

$$p^*(\pi;M) = \frac{\pi y}{f(M)} \text{ and } q^*(\pi;M) = \frac{Mf'(M)}{\pi y}.$$

In the parametric example with $f(k) = 1/\sqrt{k}$, I obtain $p^*(\pi;M) = \sqrt{M}\pi y$ and $q^*(\pi;M) = \sqrt{M}(\pi y)^{-1}$. It remains to verify if the price and quantity are consistent with the initial assumption that the supply curve is increasing at this price and the demand curve decreasing. The supply of the asset is increasing at the price as long as it does not exceed the threshold $\hat{p}$, at which mutual funds sell all of their risky asset holdings. Therefore, the candidate equilibrium is indeed on the increasing part of the supply curve if $q^*(\pi;M) < 1$. Replacing $q^*(\pi;M)$ and rearranging yields the condition $Mf'(M) < \pi y$. Lastly, it must be the case that liquidity providers' demand is downward-sloping at the price. This is the case if the price is below the asset's fundamental value, that is, if $p^*(\pi;M) < \pi y$. After replacing $p^*(\pi;M)$, this condition reduces to $1 < f(M)$.[3] Summarizing, the equilibrium configuration illustrated in Point A obtains if $Mf'(M) < \pi y$ and $1 < f(M)$.

For the second candidate equilibrium assume that mutual funds' upward-sloping supply curve intersects the horizontal part of the demand curve (Point B in Figure 2(a)). If the demand curve is flat the risky asset must trade at its fundamental value. Plugging the fundamental price in mutual funds' optimality condition Equation 1 yields

3 Note that the fire-sale discount in this candidate equilibrium is increasing in the scarcity of market liquidity. This is the cash-in-the-market pricing effect described by (Allen and Gale 1994).

Equation 8

$$p^*(\pi;M)=\pi y \quad \text{and} \quad q^*(\pi;M)=\frac{(f')^{-1}(1)}{\pi y}.$$

In the parametric example with $(f')^{-1}(1)=1$, I obtain $q^*(\pi;M)=1/(\pi y)$. To verify that this candidate equilibrium is consistent with the initial assumptions. Mutual funds' supply is indeed increasing in the asset's price if $q^*(\pi;M)<1$, which reduces to $f'(\pi y)<1$ after replacing $q^*(\pi;M)$ from Equation 8. For liquidity providers' demand to be flat, they must have sufficient liquid funds to purchase the risky asset at its fundamental price, that is, the resource constraint $pq<M$ must be satisfied. Replacing the equilibrium price and quantity from above and rearranging yields: $f'(M)<1$. Hence, this candidate equilibrium obtains if $f'(\pi y)<1$ and $f'(M)<1$ hold.

The last two candidate equilibria are located on the vertical part of the supply curve, that is, the equilibrium price must be such that mutual funds choose to sell all of their risky asset. First, suppose that mutual funds' vertical supply intersects the horizontal part of the demand curve (Point *C* in Figure 2(b)). In this case, it is immediate to obtain

$$p^*(\pi;M)=\pi y \quad \text{and} \quad q^*(\pi;M)=1.$$

At this price, mutual funds indeed sell all of their risky asset holdings if $pf'(p)>\pi y$, which after replacing $p=p^*(\pi;M)$ and rearranging yields $f'(\pi y)>1$. Liquidity providers' demand is flat if they have sufficient funds to pay the asset's fundamental price, that is, if $pq<M$, which reduces to $\pi y<M$ after replacing from above. In summary, the two conditions under which this candidate equilibrium obtains are $f'(\pi y)>1$ and $\pi y<M$.

For the last candidate equilibrium, suppose that the intersection is on the downward-sloping part of the demand curve (Point *D* in Figure 2(b)) such that I obtain

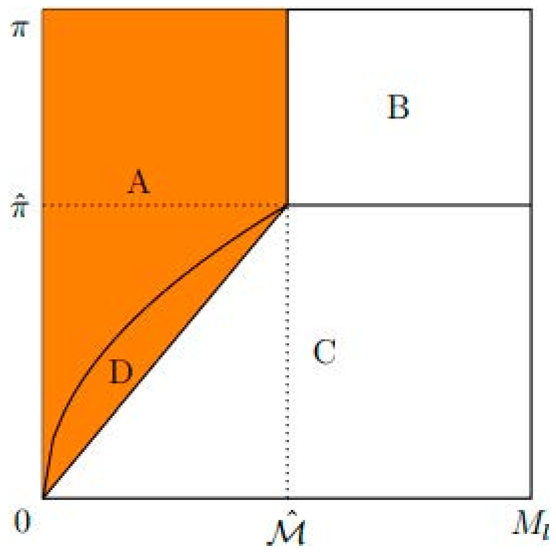
$$p^*(\pi;M)=M \quad \text{and} \quad q^*(\pi;M)=1.$$

At this price mutual funds indeed sell all of their risky asset holdings if $pf'(p)>\pi y$, which after replacing from above yields $Mf'(M)>\pi y$. Lastly, liquidity providers use all of their liquid asset holdings for risky asset purchases if the risky asset trades at fire-sale discounts, that is, if $p^*(\pi;M)<\pi y$ which can be reduced to $M<\pi y$. In summary, this candidate equilibrium obtains if $Mf'(M)>\pi y$ and $M<\pi y$.

The asset quality shock and market liquidity jointly determine which of the four possible configurations of equilibrium obtains in the asset market at date 1. Moreover, there is a unique asset market equilibrium corresponding to each combination of asset quality and market liquidity. There are two thresholds that are useful in characterizing the equilibrium regions: first, a threshold on market liquidity $\hat{M}$, implicitly defined by the equation $f'(\hat{M})=1$, such that the risky asset trades at its fundamental price regardless of the realized asset quality shock if $M>\hat{M}$. Similarly, there is a threshold on the asset quality shock $\hat{\pi}$, implicitly defined by the equation $f'(\hat{\pi}y)=1$, such that mutual funds keep some of their risky asset holdings regardless of market liquidity if $\pi > \hat{\pi}$. In the parametric example with $f'(k) = 1/\sqrt{k}$, the thresholds are $\hat{M} = 1$ and $\hat{\pi} = 1/y$.

Figure 3 illustrates the thresholds and their relevance for determining the possible configurations of equilibrium across the (M, π)-plane. There are four separate regions labeled A-D, each corresponding to one possible configuration of equilibrium labeled as Point A-D in Figure 2. This characterization illustrates that fire sales, highlighted by shaded regions, can occur only if market liquidity is scarce, that is, if $M < \hat{M}$.

Characterization of the equilibrium in the asset market FIGURE 3



Note: This figure illustrates how market liquidity M and the asset quality shock M determine the risky asset market equilibrium at date 1. Each region corresponds to an equilibrium configuration highlighted in Figure 2, with shaded regions identifying fire sales.

Moreover, Figure 3 suggests the following comparative statics of the equilibrium price and quantity of the risky asset: keeping market liquidity fixed, the traded quantity of the risky asset decreases in the asset's quality.[4] To see this more clearly, recall that if $M < \hat{M}$, mutual funds sell all of their risky asset holdings if $\pi y < Mf'(M)$ (Regions C and D). For higher realizations of the asset quality shock (Region A), mutual funds sell only $f'(M)M/(\pi y) < 1$ units of their risky asset. Similarly, if $M \geq \hat{M}$, mutual funds sell all of their risky asset holdings if $\pi < \hat{\pi}$, while they sell only a fraction of their asset holdings if $\pi \geq \hat{\pi}$. Conversely, for a fixed asset quality, a *ceteris paribus* increase in market liquidity moves the economy closer to an equilibrium without fire sales. This is because for $\pi \geq \hat{\pi}$, fire sales obtain if $M < \hat{M}$ (Region A), while fundamental pricing obtains if $M \geq \hat{M}$ (Region B). Similarly, if $\pi < \hat{\pi}$, fire sales obtain if $\pi y > M$ (Regions A and B), while fundamental pricing occurs if $\pi y \leq M$ (Region C).

4 Figure 3 additionally suggests that in an illiquid asset market ($M < \hat{M}$), a *ceteris paribus* increase in realized asset quality moves the economy closer to an equilibrium with fire sales. This feature of the equilibrium is a result of the assumptions on the production function *f*, which guarantee that mutual funds' supply of the risky asset is increasing in its price, in combination with the assumption that liquidity providers' have a fixed amount of liquid assets available for risky asset purchases.

Proposition 1 formally describes the equilibrium price and quantity of the risky asset traded as a function of its quality and market liquidity.

Proposition 1. *(Asset Market Clearing): The price and quantity of the risky asset at date 1 are uniquely determined by the realized asset quality* π *and the liquidity available in the market M. If* $M > \hat{M}$*, then*

$$p^*(\pi;M) = \pi y \quad \text{and} \quad q^*(\pi;M) = \begin{cases} 1 & \text{if } f(\pi y) \geq 1 \\ \dfrac{(f)^{-1}(1)}{\pi y} & \text{if } f(\pi y) < 1 \end{cases}.$$

If $M \leq \hat{M}$*, then*

$$p^*(\pi;M) = \begin{cases} \dfrac{\pi y}{f(M)} & \text{if } \pi y > Mf(M) \\ M & \text{if } \pi y \in [M, Mf(M)] \\ \pi y & \text{if } \pi y < M \end{cases} \quad \text{and} \quad q^*(\pi;M) = \begin{cases} \dfrac{Mf(M)}{\pi y} & \text{if } \pi y > Mf(M) \\ 1 & \text{if } \pi y \leq Mf(M) \end{cases}$$

This proposition highlights again that fire sales only occur if market liquidity is scarce, that is, if $M \leq \hat{M}$. This insight is crucial for liquidity providers' ex-ante decision on how much of the liquid asset to hold, as I discuss in the following section.

Share prices and investor redemptions. Now I determine the price of mutual fund shares at date 1 by replacing the price and quantity of the risky asset in each of the four candidate equilibria in Equation 7. In Region A, the risky asset trades at a fire-sale discount $\left(p^*(\pi;M) = \pi y / f(M)\right)$, and mutual funds sell only a fraction of their risky asset holdings $\left(q^*(\pi;M) = Mf(M)/(\pi y)\right)$, implying an interim share price of

$$s_1^*(p;\pi) = \frac{\left(1 - q^*(\pi;M)\right)\pi y}{f\left(q^*(\pi;M)p^*(\pi;M)\right)} + q^*(\pi;M)p^*(\pi;M) = \frac{\pi y}{f(M)} = p^*(\pi;M).$$

Repeating this step for the remaining possible configurations of equilibrium yields the same result: $s_1^*(p;\pi) = p^*(\pi;M)$, that is, the price of mutual fund shares at the interim date is given by the market value of the funds' risky asset holdings and varies accordingly with the realization of the aggregate shock π and the available market liquidity *M*. In other words, mutual funds mark their shares to market.

Mutual funds' budget constraint in Equation 3 determines investors' redemptions as a function of the interim share price and the equilibrium on the market for the risky asset:

$$x^*\left(s_1^*(p;\pi)\right) = \frac{q^*(\pi;M)p^*(\pi;M)}{s_1^*(p;\pi)} = q^*(\pi;M),$$

using the above result that $s_1^*(p;\pi) = p^*(\pi;M)$. Since investors' shares are priced at the market value of their mutual fund's risky asset holdings, the fund sells just as many units of the risky asset as shares are redeemed.

Ex-ante liquidity supply

This section analyzes liquidity providers' portfolio choice at date 0, which determines the aggregate market liquidity M at date 1. The first-order condition associated with liquidity providers' optimization problem, which characterizes the (private) optimal choice of liquidity, can be stated as

$$g'(w - m^*) = 1 + \frac{\partial \mathbb{E}\left[q^D\left(p^*(\pi,M);\pi,m\right)\left(\pi y - p^*(\pi,M)\right)\right]}{\partial m},$$

where $p^*(\pi;M)$ denotes the equilibrium price of the risky asset described in Proposition 1 and $q^D\left(p^*(\pi;M);\pi,m\right)$ reflects liquidity providers' demand in Equation 2 evaluated at $p = p^*(\pi;M)$. The optimal supply of liquidity balances the foregone marginal return of investing in the long-term project with the marginal (private) value of holding the liquid asset. As shown below, the marginal value of liquidity consists of the unit return of the liquid asset and an excess return when it is used to purchase the risky asset in fire sales. Liquidity providers will always choose an $m^* < w$ due to the assumption that $g'(0) = \infty$.

I begin by defining liquidity providers' (expected) payoff from holding the liquid asset at the start of date 1 conditional on the realized π:

Equation 9

$$v^p(\pi;m,M) \equiv m + q^D\left(p^*(\pi;M);\pi,m\right)\left[\pi y - p^*(\pi;M)\right],$$

Note that the (expected) profit from using the liquid asset for risky asset purchases, the second term in Equation 9, is positive only if fire sales occur. Whether the risky asset trades at fire-sale prices is independent of the investment in liquid assets m when viewed from a price-taking liquidity provider's perspective. In equilibrium, however, the risky asset's price $p^*(\pi;M)$ depends on the aggregate market liquidity M. Hence, liquidity providers' equilibrium choice of liquid asset holdings is given by a fixed point $m(M) = M$.

I begin by computing liquidity providers' marginal private value of holding the liquid asset in each of the four possible configurations of equilibrium in the risky asset market at date 1. Subsequently, I derive the expected marginal private value from investing in the liquid asset at date 0 by forming expectations about each candidate equilibrium at date 1.

If the asset market is liquid $\left(M > \hat{M}\right)$, the risky asset trades at its fundamental value and liquidity providers do not capture any surplus from purchasing it. Consequently, the marginal private benefit of liquidity is given by the liquid asset's unit return, regardless of the realized asset quality: $\partial v^p(\pi;m,M)/\partial m = 1$. From an ex-post standpoint, liquidity providers carry excess liquidity if the marginal return of their long-term project is above one, so that they incur losses from foregone long-term investment.

If market liquidity is scarce $\left(M \leq \hat{M}\right)$, the asset quality shock determines the market price of the risky asset. If the realized asset quality is low $\left(\pi y < M\right)$, the risky asset trades at its fundamental value and the liquid asset earns only the unit return. Higher realizations of the asset quality shock lead to fire sales, in which case the liquid asset earns excess returns and liquidity providers exhaust their available funds for risky asset purchases. Replacing $q^{D}\left(p^{*}(\pi;M);\pi,m\right)=m/p^{*}(\pi;M)$ in Equation 9 yields $v^{p}(\pi;m,M)=m\left[\pi y/p^{*}(\pi;M)\right]$. If the realized asset quality is intermediate $\left(\pi y \in \left[M, Mf'(M)\right]\right)$, the risky asset's fire-sale price is solely determined by the available liquidity in the asset market $\left(p^{*}(\pi;M)=M\right)$, which results in a marginal private value from liquidity of

$$\frac{\partial v^{p}(\pi;m,M)}{\partial m}=\frac{\pi y}{M}.$$

Consequently, the private value of liquidity is increasing in asset quality and the scarcity of market liquidity. If the realized asset quality is sufficiently high $\left(\pi y > Mf'(M)\right)$, the fire-sale price $p^{*}(\pi;M)=\pi y/f'(M)$ yields a marginal private value from liquidity of

$$\frac{\partial v^{p}(\pi;m,M)}{\partial m}=f'(M).$$

The expected marginal private value of liquidity at date 0 is determined by forming expectations about the marginal private value in each of the possible configurations of equilibrium at date 1=. Formally, the expected marginal value of liquidity at date 0 is $v_{0}^{p}(M)=\mathbb{E}\left[\frac{\partial v^{p}(\pi;\mathrm{m},M)}{\partial M}\right]$.

Proposition 2 summarizes liquidity providers' optimal investment in the liquid asset at date 0 and the aggregate level of liquidity in equilibrium.

Proposition 2. *(Liquidity Supply): The marginal value of holding the liquid asset at the beginning of date 1 varies with the realized asset quality shock* π *and market liquidity M:*

If $M > \hat{M}$*, the risky asset trades at its fundamental value, yielding* $\frac{\partial v^{p}(\pi;m,M)}{\partial m}=1$.

If $M < \hat{M}$*, fire sales may occur, implying potential excess returns from holding liquidity:*

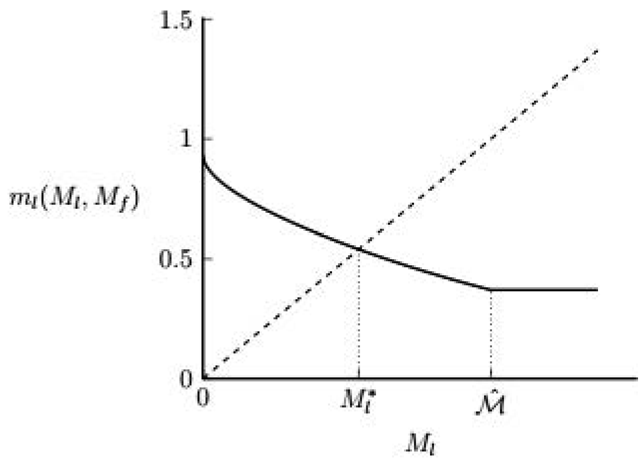
$$\frac{\partial v^{p}(\pi;m,M)}{\partial m}=\begin{cases} f'(M) & \text{if } \pi y > Mf'(M) \\ \frac{\pi y}{M} & \text{if } \pi y \in \left[M, Mf'(M)\right] \\ 1 & \text{if } \pi y < M \end{cases}.$$

The investment in the liquid asset at date 0 in the competitive equilibrium must satisfy the first-order condition $g'\left(w-m^{*}\right)=v_{0}^{p}(M)$*, which implicitly defines a value of*

m^* for each M, say m(M). In equilibrium, the aggregate level of liquidity M^* is the unique fixed-point of $m(M^*) = M^*$.

To illustrate liquidity providers' problem at date 0, I continue the parametric example with the assumptions $g(k)=2\sqrt{k}$ and $\tilde{\pi} \sim U[0,1]$. Figure 4 shows that liquidity providers' individual choice of liquidity m^* is a (weakly) declining function of aggregate liquidity *M*. Intuitively, if aggregate liquidity is low, the deviation of prices from fundamentals is high, creating a motive to hold liquidity to acquire the risky asset at fire-sale discounts. Conversely, if aggregate liquidity is high, then the expected gain from risky asset purchases is low and incentives to carry the liquid asset are minimal.

Fixed point in liquidity provision FIGURE 4



This figure illustrates how liquidity providers' optimal choice of liquidity m at date 0 varies with the aggregate liquidity M. The dashed diagonal line depicts the 45-degree line. M denotes the equilibrium level of liquidity.*

Proposition 3 summarizes the competitive equilibrium in the mutual fund industry.

Proposition 3. *(Competitive Equilibrium): In the competitive equilibrium the optimal mutual fund share price and investor redemptions at date 1 are given by* $s_1^*(p;\pi)=p^*(\pi;M)$ and $x^*(s_1^*(p;\pi))=q^*(\pi;M)$, *where* $p^*(\pi;M)$ *and* $q^*(\pi;M)$ *denote the price and quantity of the risky asset described in Proposition 1. Liquidity providers' investment in the liquid asset m* at date 0 solves* $g'(w-m(M^*))=v_0^p(M^*)$, *where* $v_0^p(M^*)$ *denotes the expected marginal private value of liquidity at date 0 and is characterized in Proposition 2.*

The proposition highlights that competitive mutual funds pass the market value of their risky asset portfolio on to investors through their share price. The implication is that when risky assets trade at fire-sale discounts, the value of mutual fund shares at date 1 drops below the fundamental value of the fund's portfolio.

Efficiency analysis

This section characterizes the efficient and constrained-efficient allocations as the solution to a social planner's problem. The results serve as benchmarks for the welfare analysis of the competitive equilibrium. The notation remains unchanged: M denotes the units of the economy's endowment w that is invested in the liquid asset at date 0, so that the investment in the long-term project g is given by $w - M$. The investment in the short-term project f at date 1 is denoted by $I(\pi)$, which may vary with the aggregate asset quality shock. The amount of the liquid asset carried from date 1 to date 2 is given by $M - I(\pi)$. The welfare function is defined as the expected unweighted sum of consumption across all agents at date 2, which can be written as

Equation 10 $$\Pi(M, I(\pi)) = g(w - M) + M + \mathbb{E}\left[\pi y + f(I(\pi)) - I(\pi)\right].$$

First best

The first-best allocation is defined as the solution to the problem of a social planner who can freely allocate the economy's endowment across the short- and long-term investment projects and the liquid asset in order to maximize social welfare. The planner only faces resource constraints when deciding how to invest the endowment w.

The *first-best allocation* is a pair of investment decisions $(M, I(\pi))$ that maximizes social welfare in Equation 10 subject to the resource constraint at date 0

$$M \leq w,$$

the resource constraint at date 1

$$I(\pi) \leq M$$

and the feasibility constraints

$$M, I(\pi) \geq 0.$$

The resource constraint at date 0 ensures that the ex-ante investment in the liquid asset does not exceed the economy's endowment. The resource constraint at date 1 states that the interim investment in the short-term project f can only be funded with the available liquid asset. The feasibility constraints highlight that short selling the liquid asset is not permitted. The planner's choice variables are denoted by uppercase letters to reflect the fact that she is choosing aggregate quantities. Note that the economy's endowment of the risky asset plays no role in the planner's allocation problem. The risky asset is in fixed unit supply at date 0 and generates no payoffs before it matures at date 2. Therefore the planner's allocation problem is not affected by the asset quality shock and we simply write $I(\pi)=I$ to denote the investment in the short-term project at date 1.

The first-order conditions associated with the planner's problem characterize the interior solution, which equates the marginal returns from the short and long-term project with the unit return of the liquid asset:

$$\begin{aligned}\frac{\partial\Pi(M,I)}{\partial M} &= -g'(w-M)+1=0\\ \frac{\partial\Pi(M,I)}{\partial I} &= f'(I)-1=0\end{aligned}$$

Whenever we have an interior solution, the first-best allocation is given by the solution to these optimality conditions, which we denote by $(\overline{M},\overline{I})$. The interior solution obtains if $(\overline{M},\overline{I})$ satisfy the resource and feasibility constraints, which is the case whenever $\overline{M}\geq\overline{I}$. This implies a threshold $\hat{w}$, implicitly defined by the equation

$$\overline{M}(\hat{w})=\overline{I},$$

such that the first-best allocation is determined by the first-order conditions if $w\geq\hat{w}$. If $w<\hat{w}$, there are not enough resources available to equalize the marginal returns of the short and long-term projects to the unit return of the liquid asset. Consequently, the planner equalizes the marginal returns of the short and long-term projects, which remain above the liquid asset's unit return. The planner invests just enough into the liquid asset at date 0 to achieve this and does not carry any of the liquid asset after date 1.

Proposition 4 summarizes the first-best allocation.

Proposition 4. *(First Best): If $w\geq\hat{w}$, the first best levels of ex-ante liquidity provision $M^{FB}$and investment in the short-term project I^{FB} (which does not depend on π) solve $f'\left(I^{FB}\right)=g'\left(w-M^{FB}\right)=1$, and the amount of the liquid asset held until date 2 is given by $M^{FB}-I^{FB}$. If $w<\hat{w}$, the planner sets $M^{FB}=I^{FB}$ to solve $g'\left(w-M^{FB}\right)=f'\left(M^{FB}\right)>1$ and carries none of the liquid asset until date 2.*

In the parametric example, it is straightforward to show that the threshold on the economy's endowment is $\hat{w}=2$. If $w\geq2$, the first-best allocation is given by $M^{FB}=w-1$ and $I^{FB}=1$. If $w<2$, the first-best allocation is $M^{FB}=I^{FB}=w/2$.

Second best

This section analyzes a version of the social planner's problem in the spirit of Stiglitz (1982), in which the planner can only determine the investment in the liquid asset at date 0. The constrained planner faces the same constraints as liquidity providers and leaves all decisions at date 1 to private agents, respecting that the price of the risky asset is determined as in Section 3. However, in contrast to infinitesimal liquidity providers in the competitive equilibrium, the constrained planner internalizes the effect of the initial liquidity choice on the equilibrium in the asset market. In particular, the planner anticipates that the investment in the short-term technology at date 1 is determined by how many units of the consumption good investors raise by selling the risky asset:

Equation 11 $$I(\pi;M) = p^*(\pi;M)q^*(\pi;M),$$

where $p^*(\pi;M)$ and $q^*(\pi;M)$ denote, respectively, the equilibrium price and the quantity of risky assets in the competitive equilibrium described in Proposition 1.

The constrained planner's investment in the liquid asset M at date 0 maximizes social welfare in Equation 10 subject to the investment in the short-term project being determined by agents trading on the competitive asset market as in Equation 11. The planner's (expected) payoff from holding the liquid asset at the start of date 1, conditional on the realized π, is

$$v^s(\pi;M) = M + f\left(p^*(\pi;M)q^*(\pi;M)\right) - p^*(\pi;M)q^*(\pi;M).$$

The expected marginal social value of liquidity at date 0 is thus

$$v_0^s(M) = \mathbb{E}\left[\frac{\partial v^p(\pi;M)}{\partial M}\right],$$

and the interior solution to the constrained planner's problem is characterized by

$$g'(w-M) = v_0^s(M).$$

The constrained planner invests in the liquid asset until the marginal cost of this investment, the foregone marginal return of the long-term project, equals its marginal value. The marginal social value of liquidity consists of the liquid asset's unit return and the potential of increasing the investment in the short-term project f.

I begin by determining the marginal social value of liquidity in each of the four possible equilibrium configurations in the asset market at date 1. Subsequently, forming expectations about the candidate equilibria yields the expected marginal social value of investing in the liquid asset at date 0. If the asset market is liquid $\left(M > \hat{M}\right)$, the risky asset trades at its fundamental price and the planner's choice of liquidity has no marginal impact on the investment in the short-term project. Consequently, the marginal social value of liquidity is given by the liquid asset's unit return: $\partial v^s(\pi;M)/\partial M = 1$.

Instead, if $M < \hat{M}$, liquidity in the asset market is scarce and fire sales may occur. For low realizations of the asset quality shock $(\pi y < M)$, investors sell all of their risky asset holdings at the fundamental price, thereby raising $p^*(\pi;M)q^*(\pi;M) = \pi y$ units of the consumption good. In this situation, a marginal increase in available liquid funds has no impact on the investment in the short-term project: $\partial v^s(\pi;M)/\partial M = 1$. However, if the asset quality shock is sufficiently high $(\pi y > M)$, fire sales occur and the investment in the interim technology is solely determined by market liquidity: $p^*(\pi;M)q^*(\pi;M)=M$. Consequently, a marginal increase in available liquid funds expands the productive investment in the short-term project, and liquidity earns a premium

$$\frac{\partial v^s(\pi;M)}{\partial M}=f'(M)>1.$$

Proposition 5 summarizes how the marginal social value of holding the liquid asset at date 1 varies with the aggregate state of the economy.

Proposition 5. *(Social Value of Liquidity): The marginal social value of the liquid asset at date 1 varies with the realized asset quality shock and market liquidity: If $M \geq \hat{M}$, the risky asset trades at its fundamental value, yielding $\partial v^s(\pi;M)/\partial M=1$. If $M<\hat{M}$, fire sales may occur, implying potential excess returns from holding liquidity:*

$$\frac{\partial v^s(\pi;M)}{\partial M}=\begin{cases} f'(M) & \text{if } \pi y \geq M \\ 1 & \text{if } \pi y < M \end{cases}.$$

This result indicates that the marginal social value of liquidity may deviate from its private value, the implications of which are studied in the following subsection.

Inefficient liquidity supply

This section compares the provision of liquidity in the competitive equilibrium to the first and second-best benchmarks. Proposition 1 summarizes the key result of this section.

Proposition 6. *(Efficiency of Liquidity Supply): Whether liquidity supply in the competitive equilibrium is (constrained) efficient depends on the relative scarcity of liquidity providers' endowment w. If $w<\hat{w}$, $M^{CE}<M^{SB}<M^{FB}$ where M^{CE}, M^{SB}, and M^{FB} denote, respectively, the supply of liquidity in the competitive equilibrium and the second and first-best benchmarks. If $w\geq\hat{w}$, $M^{CE}=M^{SB}=M^{FB}$.*

Proof. The proof of Proposition 6 proceeds in two steps. In the first step, I show that if $w\geq\hat{w}$, liquidity provision in the competitive equilibrium coincides with the first and second-best benchmarks. The second step of the proof shows that if $w<\hat{w}$, liquidity provision in the competitive equilibrium is below the constrained efficient benchmark, which in turn is below the first best.

Step 1: Efficient liquidity provision for $w\geq\hat{w}$. Assume that the asset market is liquid ($M>\hat{M}$), such that the risky asset trades at its fundamental value, regardless of the realized asset quality shock. Then the first-order conditions determining liquidity provision M in the competitive equilibrium and the second best reduce to

$$g'\left(w-M^*\right)=1,$$

which is identical to the optimality condition determining the first-best level of liquidity supply. The implied level of liquidity provision $M=M^*$ is consistent with the initial assumption of a liquid asset market if $M^* \geq \hat{M}$. Solving for the explicit expressions of M^* and $\hat{M}$ shows that the condition is equivalent to $w \geq \hat{w}$. Hence, if $w \geq \hat{w}$, liquidity supply in the competitive equilibrium and second best attain the first best.

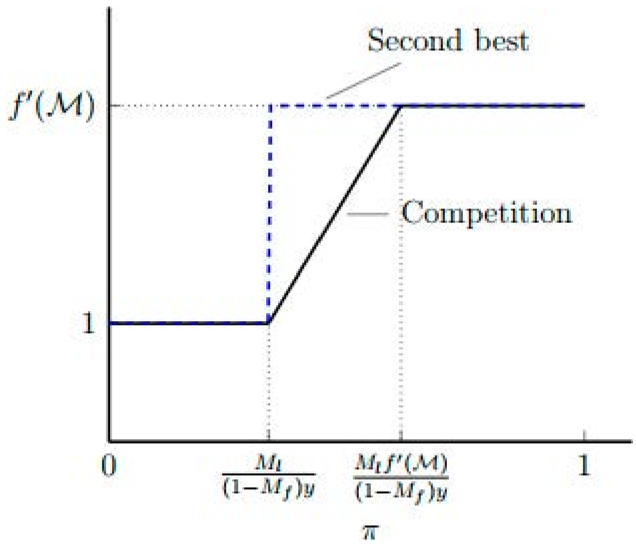
Step 2: Inefficient liquidity provision if $w < \hat{w}$. In order to show that liquidity supply in the competitive equilibrium is below the constrained planner's choice, it suffices to compare the optimality condition of competitive liquidity providers to the one of the constrained planner. The opportunity cost of investing in the liquid asset, the foregone return from the long-term project g, is the same for the constrained planner and competitive liquidity providers. It follows that any differences in liquidity supply arise due to differences in the marginal private and social value of liquidity.

Comparing the marginal private and social value of liquidity at date 1, derived in Propositions 2 and 5 respectively, shows that they differ if the competitive equilibrium features $M^{CE} < \hat{M}$. Their difference can be expressed as

$$\frac{\partial v^s(\pi;M)}{\partial M} - \frac{\partial v^p(\pi;M)}{\partial M} = \begin{cases} f'(M) - \frac{\pi y}{M} > 0 & \text{if } \pi y \in [M, Mf'(M)] \\ 0 & \text{otherwise} \end{cases},$$

which is illustrated in Figure 5.

Marginal value of liquidity providers' investment in the liquid asset FIGURE 5



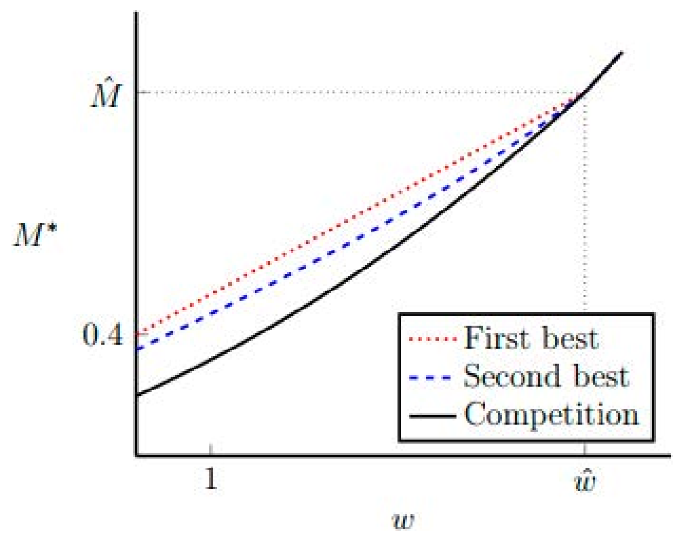
Note: This figure illustrates how the marginal value from the liquid asset for competitive liquidity providers (solid line) and the constrained social planner (dashed line) varies with the realized asset quality π at date 1 when the asset market is illiquid ($M < \hat{M}$).

The figure shows that the marginal social value of liquidity is weakly higher than its private value to liquidity providers, and if the realized asset quality shock satisfies $\pi \in [M/y, Mf'(M)/y]$, the social value of liquidity is strictly above its private value. Lastly, recall that if $w < \hat{w}$, liquidity in the competitive equilibrium indeed satisfies

$M^{CE} < \hat{M}$. It follows that liquidity provision in the competitive equilibrium is below the constrained efficient benchmark.

Figure 6 illustrates the optimal liquidity provision in the first and second best as well as in the competitive equilibrium as a function of the liquidity providers' endowment w. For any $w < \hat{w}$, the constrained planner's liquidity choice is below the first best due to the ex-ante uncertainty about the asset quality shock, which may restrict the scale of the investment in the short-term project f by limiting how many units of the consumption good investors can raise in the asset market. If the risky asset is of low quality, investors sell all of their risky asset holdings but fail to raise sufficient funds to achieve the first-best investment in the short-term project.

Inefficient liquidity provision in competitive equilibrium FIGURE 6



Note: This figure illustrates how the optimal liquidity supply in the first best (dotted line), second best (dashed line), and competitive equilibrium (solid line) varies with the economy's endowment w.

In the competitive equilibrium, liquidity supply is even below the constrained efficient benchmark. This is due to a pecuniary externality in liquidity providers' ex-ante liquidity choice: because markets are incomplete, risky asset purchases can only be financed with the liquid asset in liquidity providers' portfolio. This leads to states of the world at date 1 in which the lack of available market liquidity restricts mutual funds' revenue from risky asset sales. To see this clearly, recall that equilibrium region D features $p^*(\pi;M)q^*(\pi;M) = M$ and $p^*(\pi;M)q^*(\pi;M) = s_1^*(p;\pi)x^*\left(s_1^*(p;\pi)\right)$, that is mutual funds cannot pay redeeming investors more than the available cash in the market M to finance their investment in the short-term project f. In other words liquidity providers do not fully account for the marginal social value of their liquidity holdings which increase investors' investment in the short-term project. The underprovision of liquidity by private agents is thus a result of liquidity's public good character as in (Bhattacharya and Gale, 2011) and leads to an inefficiently high likelihood of fire sales. In order to isolate the role of the pecuniary externality in generating the discussed efficiencies, I develop a simplified model which abstracts from mutual funds' optimal contracting problem in Appendix 7 and show that it replicates the key results discussed above.

Mutual fund regulation

This section studies the effects of regulatory policies, namely a mandatory liquidity buffer for mutual funds and redemption gates. It also sheds light on whether competitive mutual funds build up efficient liquidity buffers in the absence of a regulatory requirement. To focus on the parameter setting in which inefficiencies arise, I assume $w < \hat{w}$ going forward.

Liquidity requirements

This section explores the effect of a mandatory liquidity buffer requiring mutual funds to hold at least a fraction of their portfolio in the form of the liquid asset. For simplicity, I assume that the liquidity buffer is always binding such that funds never voluntarily hold more liquid assets than they are required to. This assumption is strengthened by the following subsection, which shows that competitive funds indeed choose to hold less liquidity than socially optimal. Formally, the liquidity buffer requires each mutual fund to invest the unit endowment collected from investors at date 0 in a portfolio $(m_f, 1 - m_f)$, where m_f denotes the fund's liquid asset holdings. I distinguish a mutual fund's liquidity holdings m_f from liquidity supply by a liquidity provider m_l (previously simply denoted m). The baseline model discussed in the previous sections obtains by setting the liquidity requirement to zero ($m_f = 0$) such that the mutual fund is fully invested in the risky asset. Since the liquidity buffer reduces the fund's risky asset holdings, it changes the supply of the risky asset at date 1 and consequently the equilibrium on the asset market.

Supply of the risky asset at $t = 1$. The representative mutual fund enters date 1 with a portfolio $(m_f, 1 - m_f)$ and seeks to maximize investors' expected date 2 consumption. To do so, the fund chooses an interim share price s_1 and accommodates redemptions by i) selling q units of the risky asset at market price p and ii) using the liquid asset m_f in its portfolio. To simplify the exposition, I assume that the mutual fund accommodates redemptions first using the liquid asset before resorting to risky asset sales.[5] Then it is straightforward to show that the fund's problem of choosing a pair (s_1, q) subject to the same constraints as in the baseline model can again be reduced to choosing the optimal level of risky asset sales to maximize investors' expected consumption, that is,

$$\max_{q \in [0, 1-m_f]} \left\{ f\left(qp + m_f\right) + \left(1 - m_f - q\right)\pi y \right\}.$$

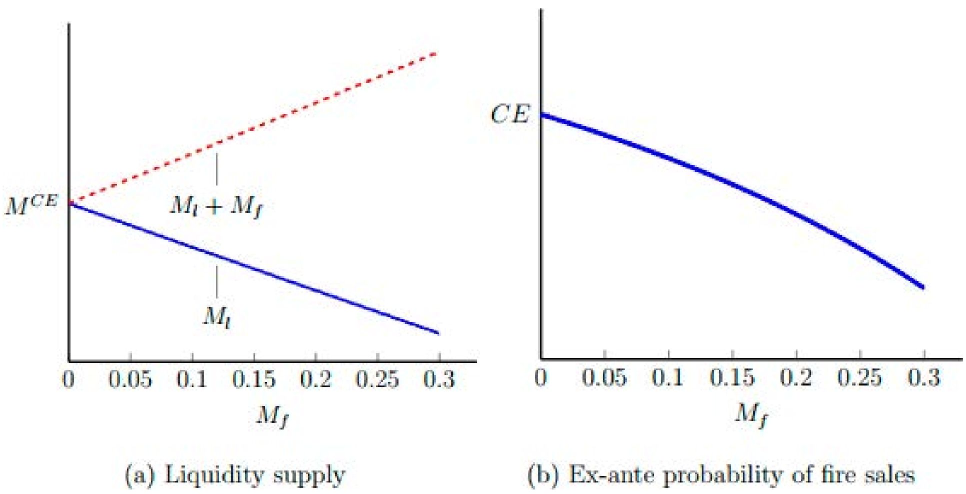
5 Appendix 8 shows that investors optimally invest all of their liquid asset holdings in the short-term project at date before resorting to risky asset sales under very mild assumptions. Ma, Xiao, and Zeng (2020) provide empirical evidence showing that mutual funds indeed follow such a pecking order.

The problem is very similar to the one presented in Section 2, apart from the fact that the mutual fund owns only $1 - m_f$ units of the risky asset. The derivation of the regulated equilibrium is therefore relegated to Appendix 8. Here I focus on presenting the key results of the regulated equilibrium and denote the *aggregate* amounts of liquid asset holdings by liquidity providers and mutual funds by M_l and M_f, respectively.

The liquidity buffer reduces the equilibrium regions in which fire sales occur at date 1. Intuitively, for a given level of market liquidity $M_l + M_f$, the likelihood of fire-sales declines because the liquidity buffer reduces mutual funds' need for risky asset sales. However, the decreased likelihood of fire sales has a negative effect on liquidity providers' incentives to invest in the liquid asset: the expected return from investing in the liquid asset declines since the opportunities to earn excess returns on liquidity in fire sales diminish. The net effect of liquidity requirements in equilibrium depends on the degree to which the liquidity requirement crowds out liquidity provision by private agents.

Figure 7(a) illustrates the effect of the liquidity requirement on liquidity providers' liquidity supply and market liquidity. It shows that, starting from the unregulated equilibrium with $M_f = 0$ and $M_l = M^{CE}$, increasing the liquidity buffer reduces endogenous liquidity supply. In other words, liquidity providers' ex-ante investment in the liquid asset declines as mutual funds' liquidity buffer grows.[6] However, Figure 7(a) shows that the endogenous supply of liquidity decreases less than one-to-one with the liquidity buffer. Therefore, market liquidity in the regulated equilibrium increases compared to the unregulated equilibrium. Due to the increase in market liquidity, the liquidity requirement leads to a reduction in the ex-ante probability of fire sales compared to the unregulated competitive benchmark as depicted in Figure 7(b).

Equilibrium effects of a liquidity requirement FIGURE 7



Note: This figure illustrates the effect of the liquidity requirement M_f on liquidity supply M_l and asset market liquidity $M_f + M_l$ (Panel a) and the likelihood of fire sales (Panel b). The unregulated competitive equilibrium obtains for $M_f = 0$.

6 Appendix 8 shows formally that the expected marginal private value of liquidity is strictly decreasing in the liquidity buffer of mutual funds.

Endogenous mutual fund liquidity buffers

The positive effects of a liquidity requirement for mutual funds raise the question whether competitive funds would hold efficient liquidity buffers if regulation did not mandate this. This section studies an extension of the baseline model in which mutual funds choose how to invest the funds collected from investors at date 0 in a portfolio of the risky asset and the liquid asset. Subsequently, I compare the competitive equilibrium of this model to a constrained efficient benchmark in which a social planner chooses the initial investment in the liquid asset by liquidity providers and mutual funds.

Deriving the competitive equilibrium begins with an analysis of the equilibrium on the asset market at date 1 given arbitrary levels of asset quality π and liquidity in the hands of mutual funds M_f and liquidity providers M_l. This is identical to the analysis of the asset market equilibrium under liquidity regulation derived in Appendix 8. Therefore, liquidity providers' initial portfolio choice problem at date 0 remains unchanged too and I can focus directly on mutual funds' portfolio choice problem below.

Mutual funds' liquidity decision at date 0. At date 0, each mutual fund collects investors' unit endowment of the consumption good and invests it on their behalf in a portfolio consisting of two financial assets: first m_f units of the liquid asset, which at date 1 are used to accommodate redemptions, and $1 - m_f$ units of the risky asset $\tilde{y}$, which may be sold on the risky asset market at date 1 or be held until maturity at date 2.

The representative mutual fund chooses the investment in the liquid asset to maximize its investors' expected consumption at date 2, that is

$$\max_{m_f \in [0,1]} \left\{ \mathbb{E}\left[f\left(qp + m_f\right) + \left(1 - m_f - q\right)\pi y \right] \right\}.$$

The first-order condition associated with the fund's optimization problem characterizes its (private) optimal choice of liquid asset holdings

$$\bar{\pi} y = \mathbb{E}\left[\frac{\partial \left\{ f\left(q^S\left(p^*; \pi, m_f, \bar{M}\right) p^* + m_f \right) - q^S\left(p^*; \pi, m_f, \bar{M}\right) \pi y \right\}}{\partial m_f} \right],$$

where $\bar{\pi} \equiv \mathbb{E}[\pi], \bar{M} \equiv (M_f, M_l)$ and $q^S(p^*; \pi, m_f, \bar{M})$ reflects the fund's supply of the risky asset evaluated at the equilibrium price $p^* = p^*(\pi; \bar{M})$, which now depends on aggregate liquidity in the hands of both mutual funds M_f and liquidity providers M_l.

The mutual fund's optimal investment in the liquid asset balances the foregone expected return of investing in the risky asset with the marginal (private) value of holding the liquid asset. The marginal private value of liquidity consists of the expected return of accommodating redemptions, and thereby increasing investors' investment in the short-term project f, with the liquid asset instead of resorting to risky asset (fire) sales.

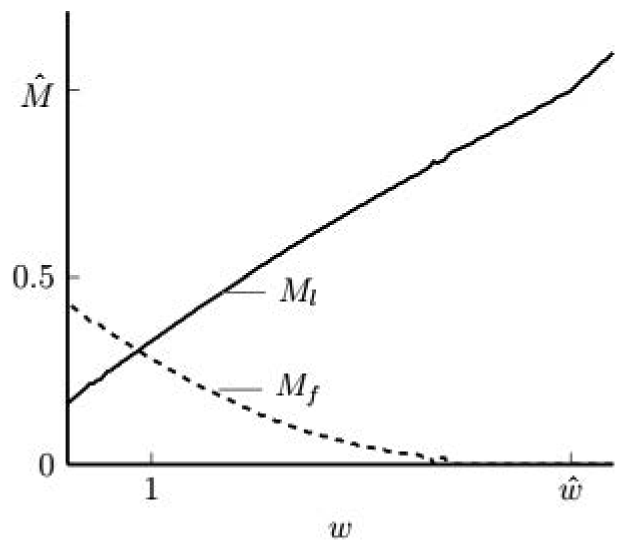
To derive the expected marginal value of mutual fund liquidity, I define the fund's payoff from holding the liquid asset at date 1 conditional on the realized π:

$$\rho^{p}(\pi;m_{f},\overline{M}) \equiv f(q^{S}(p^{*};\pi,m_{f},\overline{M})p^{*}+m_{f})-q^{S}(p^{*};\pi,m_{f},\overline{M})\pi y.$$

The remaining steps to solve for the competitive equilibrium with endogenous mutual fund liquidity follow the same procedure as in the baseline model and are based on calculating the marginal private value of mutual fund liquidity $\rho^{p'}(\pi;m_{f},\overline{M}) \equiv \partial\rho^{p}(\pi;m_{f},\overline{M})/\partial m_{f}$ in each candidate equilibrium at date 1 using the respective equilibrium price of the risky asset. The details of these derivations are relegated to Appendix 9.1. The remainder of this section presents the key results of this model extension.

Figure 8 illustrates how liquidity provision in the competitive equilibrium with endogenous mutual fund liquidity varies with the size of liquidity providers' endowment w. If liquidity providers' endowment is small, they lack the resources to invest sufficiently in the long-term illiquid project g to equalize its marginal return with the expected return of the liquid asset. Consequently, their portfolio is tilted toward the long-term, high-marginal return project, and they hold little of the liquid asset. Mutual funds anticipate this and hold substantial liquidity buffers to insulate their investors from potential fire sales of the risky asset at date 1. For larger parameter values of the endowment w, liquidity providers' investment in the long-term project increases, which drives down its marginal return, so that their portfolio increasingly tilts toward the liquid asset. In response, mutual funds' liquidity buffers shrink substantially. Interestingly, mutual funds do not hold sufficient liquidity buffers to completely rule out fire sales in equilibrium, that is, for any $w < \hat{w}$, the equilibrium features $M_f + M_l < \hat{M}$.

Liquidity provision with endogenous mutual fund liquidity buffers FIGURE 8



Note: This figure illustrates how liquid asset holdings by liquidity providers M_l (solid line) and mutual funds M_f (dashed line) vary with liquidity providers' endowment w.

Second best with endogenous mutual fund liquidity. This section derives a constrained efficient benchmark for the extended model with endogenous mutual fund liquidity. In this second best, the constrained planner determines the investment in the liquid asset by liquidity providers M_l and mutual funds M_f at date 0 to maximize social welfare at date 2, that is,

$$\max_{\substack{M_f \in [0,1] \\ M_l [0,w]}} \{g(w - M_l) + M_l + E[(1 - M_f)\pi y + f(p^*(\pi, \bar{M})q^*(\pi, \bar{M}) + M_f) - p^*(\pi, \bar{M})q^*(\pi, \bar{M})]\}.$$

Contrary to infinitesimal mutual funds, the planner accounts for the effect of her liquidity choices on the equilibrium price of the risky asset at date 1. The optimality condition determining the planner's choice of mutual fund liquidity M_f,

$$\bar{\pi} y = \mathbb{E}\left[\frac{\partial\left\{f\left(p^*\left(\pi, \bar{M}\right)q^*\left(\pi, \bar{M}\right) + M_f\right) - p^*\left(\pi, \bar{M}\right)q^*\left(\pi, \bar{M}\right)\right\}}{\partial M_f}\right],$$

balances the foregone expected return of investing in the risky asset $\bar{\pi} y$ with the marginal (social) value of holding the liquid asset. The planner's (expected) social payoff from mutual fund liquidity at the start of date 1 conditional on the realized π is:

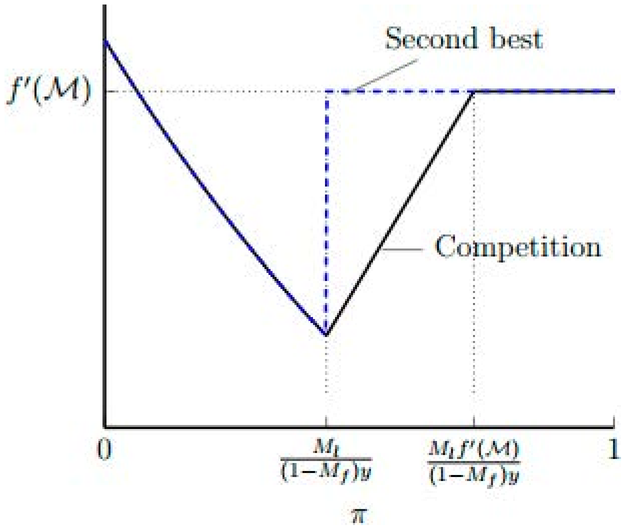
$$\rho^s\left(\pi; \bar{M}\right) = f\left(p^*\left(\pi, \bar{M}\right)q^*\left(\pi, \bar{M}\right) + M_f\right) - p^*\left(\pi, \bar{M}\right)q^*\left(\pi, \bar{M}\right).$$

The derivation of the constrained efficient benchmark is relegated to Appendix 9.2.

Efficiency of endogenous liquidity buffers. To assess whether competitive mutual funds' liquidity buffers are constrained efficient, I compare the optimality conditions determining mutual fund liquidity in the competitive equilibrium and the second-best benchmark. The marginal cost of investing in the liquid asset, the foregone expected returns from investing in the risky asset, is the same for competitive mutual funds and the constrained planner. Potential inefficiencies in mutual funds' liquidity buffers are thus determined by differences in the private marginal value of mutual fund liquidity at date 1, $\partial \rho^p(\pi; \bar{M}) / \partial M_f$, and the social value, $\partial \rho^s(\pi; \bar{M}) / \partial M_f$.

Figure 9 illustrates that the marginal social value of mutual fund liquidity is weakly higher than its private one. Moreover, if the realized asset quality shock satisfies $\pi \in \left[\frac{M_l}{(1 - M_f)y}, \frac{M_l f'(M_l + M_f)}{(1 - M_f)y}\right]$, which corresponds to asset market candidate equilibrium region D, the social value of mutual fund liquidity is strictly above its private value. Mutual funds' endogenous liquidity holdings are thus below the second-best benchmark, resulting in an equilibrium with an inefficiently high likelihood of fire sales.

Marginal value of mutual fund liquidity FIGURE 9



Note: This figure illustrates how the marginal value of the liquid asset in mutual funds' portfolio varies with the realized asset quality π. The figure contrasts the private value for mutual funds (solid line) with the social value for the constrained planer (dashed line).

Redemption gates

This section explores the effect of redemption gates, which limit investor redemptions in times of market distress. I model redemption gates as a constraint on the sales of the risky asset so that its price cannot drop below a proportion $1-\delta$ of its fundamental value πy. The parameter δ can thus be understood as the threshold for market distress at which redemption gates are triggered. Since competitive mutual funds pass the market value of their portfolio on to investors via their share price, that is, because $s_1^*(p;\pi) = p^*(\pi;M)$ in the baseline model, redemption gates imply that the regulated equilibrium features

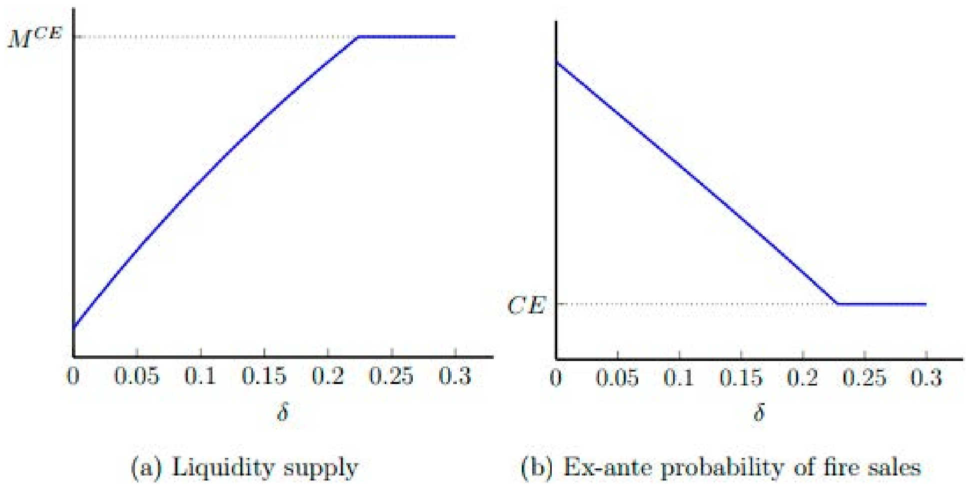
$$s_1^R(\pi;M) = \max\{p^*(\pi;M),(1-\delta)\pi y\}.$$

The derivation of the equilibrium with redemption gates is relegated to Appendix 10.

Before illustrating the effects of redemption gates, it is worth highlighting how their effects differ from liquidity requirements for mutual funds. At date 1, liquidity buffers allow mutual funds to insulate their investors from states of the world in which mild fire sales would occur in the unregulated equilibrium. In those states, mutual funds' liquid asset holdings reduce the need for risky asset sales, thereby avoiding exhausting the available market liquidity and triggering fire sales . Figure D.4 in Appendix 10 shows that redemption gates instead aim to insulate investors from states of the world with large fire-sale discounts, as they are only triggered when the discount crosses the threshold δ.

Figure 10(a) illustrates the effect of redemption gates on asset market liquidity. High values of δ imply that redemption gates are triggered only when fire sales are very severe. Consequently, liquidity supply in the regulated equilibrium coincides with the unregulated competitive benchmark M^{CE} for high values of δ. As δ declines, the threshold for activating redemption gates becomes smaller and limits the severity of potential fire sales. This diminishes expected rents from holding liquidity for risky asset purchases and leads to a reduction of liquidity supply. In the limit $(\delta = 0)$, fire sales are ruled out and market liquidity is substantially lower than in the competitive unregulated equilibrium. Figure 10(b) shows that the decrease in market liquidity due to a tightening of redemption gates (lowering δ) results in an increase in the ex-ante probability of fire sales.

Equilibrium effects of redemption gates FIGURE 10



Note: This figure illustrates the effect of redemption gates on liquidity supply M_1 (Panel a) and the probability of fire sales (Panel b). Reducing the threshold δ from the unregulated benchmark $(\delta = 1)$ implies that gates are activated more often, that is the policy tightens.

This result implies that a sufficiently low threshold for triggering redemption gates leads to a reduction in asset market liquidity and more (mild) fire sales. The increased likelihood of such fire sales is sufficient to outweigh the gains from insulating investors from the most severe fire sales, as redemption gates lower social welfare (see Appendix 10 for details).

Conclusion

This paper studied the implications of liquidity risk for open-ended mutual funds investing in imperfectly liquid assets. The analysis is based on an equilibrium model of investor redemptions in mutual funds whose assets are traded on a competitive asset market. Redemptions lead mutual funds to sell some of their assets, which may lead to fire sales due to a cash-in-the-market pricing effect as in Allen and Gale (1994). Liquidity supply to the underlying asset market is endogenously driven by arbitrageurs' expectation of purchasing undervalued assets in fire sales. Arbitrageurs fail to internalize the impact of their liquidity supply on asset prices due to a pecuniary externality resulting from market incompleteness, leading to a (constrained) inefficient equilibrium.

The theoretical framework developed in this paper can help policymakers assess the impact of various regulatory reform proposals for the mutual fund industry. I showcased the framework's flexibility with an analysis of a mandatory liquidity buffer that requires mutual funds to hold liquid assets, and redemption gates, which restrict investor redemptions during fire-sale episodes. The results showed that the evaluation of reforms must account for the endogenous response of asset market liquidity to regulation: policies lowering mutual funds' need to liquidate assets (in fire sales) disincentivize the build up of arbitrage capital and therefore adversely affect liquidity supply to the underlying asset market. While liquidity requirements leave sufficient incentives to buildup arbitrage capital, under redemption gates this negative effect is strong enough to outweigh the benefits of regulation and results in less liquid asset markets and a higher incidence of fire sales.

The results illustrate the importance of equilibrium effects of liquidity risk regulation in the mutual fund industry and a welfare trade-off: regulation benefits mutual funds' investors, who are being insulated from potential fire sales, at the expense of liquidity providers in the underlying asset market, whose expected returns from holding liquidity decline as regulation makes fire sales less likely. Quantitative evaluations of this trade-off, such as those needed to determine optimal levels of liquidity requirements, are outside the scope of the parsimonious framework developed in this paper. However, the connection between mutual funds and liquidity in the underlying asset market highlighted by this framework can guide the development of quantitative models in future research.

Appendix

Pecuniary externalities in a simplified model

This section develops a simplified version of the baseline model to highlight the key modeling ingredients needed to generate inefficiencies. The simplified model abstracts from mutual funds as financial intermediaries and focuses on the interaction between investors and liquidity providers on the risky asset market. This may be interpreted as a situation in which limited market participation by investors is not a first-order concern.

Consider an economy with three dates indexed by $t \in \{0,1,2\}$. There is a single, homogeneous consumption good which serves as the numéraire. There are two classes of risk-neutral agents: investors and liquidity providers. All agents consume only at the final date and do not discount their future consumption.

Investors. There is a continuum of investors with measure one. At date 0 each investor is endowed with one unit of a divisible risky long-term asset with payoff at date 2 given by $\tilde{y}$, which may be equal to 0 with probability $1-\pi$ or $y>0$ with probability π. The success probability π is an aggregate asset quality shock that is observed at date 1. From the perspective of date 0 the shock has a cumulative distribution function $H(\pi)$. After the realization of the asset quality shock each investor gains access to a short-term investment project that transforms k units of the consumption good at date 1 into $f(k)$ units of the consumption good at date 2. The production function $f(k)$ is increasing and concave and satisfies $f'(0)=\infty$ and $kf''(k)+f'(k)>0$.

Since the risky asset does not generate any payoff at date 1, investors raise funds to invest in the short-term project by selling some of their risky asset in a competitive market. Let p denote the price of one unit of the risky asset at date 1 (which will be a function of the realization of the aggregate shock π). Selling q units of the risky asset yields pq units of the consumption good at date 1 to invest in the short-term project.

The representative investor chooses her supply of the risky asset to maximize her consumption at date 2, that is,

$$\max_{q\in[0,1]} \left\{ f\left(pq\right)+\left(1-q\right)\pi y \right\}.$$

Note that the investor's problem in this simplified model is identical to the mutual fund's problem of choosing the optimal asset liquidation policy in Section 2. Consequently, the optimal supply of the risky asset is denoted $q^S\left(p;\pi,m\right)$ and given by Equation 1.

Liquidity providers. There is a continuum of liquidity providers, each endowed with w units of the consumption good at date 0. The problem of liquidity providers is identical to the one presented in section 2. They maximize their expected consumption at date 2 by choosing i) the investment in the liquid asset m at date 0 and ii) how many units of the risky asset q^D to purchase at date 1, which will depend on the aggregate state π. The representative liquidity provider chooses a pair $\left(m,q^D\right)$ to solve

$$\begin{aligned} \max_{m,q^D} \quad & \left\{g(w-m)+m+\mathbb{E}\left[q^D\left(\tilde{\pi}y-p\right)\right]\right\} \\ \text{subject to} \quad & m \leq w \\ & q^D \leq \frac{m}{p} \\ & m,q^D \geq 0 \end{aligned}$$

As liquidity providers' problem remains unchanged with respect to Section 2, the optimal demand for risky assets is denoted $q^D\left(p;\pi,m\right)$ and given by Equation 2.

Competitive equilibrium. Since the supply and demand for the risky asset remain unchanged with respect to the full model presented in Section 2, the equilibrium price and the quantity of the risky asset traded are those described in Proposition 1. Consequently, the optimal supply of liquidity at date 0 in the competitive equilibrium is described in Proposition 2, which leads to the inefficiencies discussed in Section 4.

Equilibrium with liquidity requirements

In this section I derive the equilibrium under a liquidity requirement for mutual funds by repeating the same steps as in the unregulated competitive equilibrium. Due to the repetitive nature of the exercise, comments are intentionally kept brief.

First, I motivate the assumption that mutual funds pay redeeming investors first with their liquid asset holdings before resorting to risky asset sales. Consider a representative mutual fund entering date 1 with a portfolio $(m_f,1-m_f)$. The fund accommodates redemptions by i) selling q units of the risky asset at market price p and ii) using $l \in [0,m_f]$ units of the liquid asset holdings. Thus, the representative fund chooses a tuple (q,l) to solve

$$\max_{q\in[0,1-m_f],l\in[0,m_f]} \{f(qp+l)+(1-m_f-q)\pi y+m_f-l\}.$$

The first-order conditions to this problem are

$$\begin{aligned} \frac{\partial}{\partial q} &= f'(qp+l)p-\pi y=0 \\ \frac{\partial}{\partial l} &= f'(qp+l)-1=0. \end{aligned}$$

The optimality conditions show that the opportunity cost of selling one unit of the risky asset to invest is $\pi y/p$, while the opportunity cost of investing the liquid asset is 1. Clearly, if the risky asset is sold in fire sales $(p<\pi y)$ we have $\frac{\pi y}{p}>1$ so that investing the liquid asset strictly dominates. Instead, if the risky asset is sold at its fundamental price ($p = \pi y$), both funding modes carry the same opportunity cost.

Case 1: $p < \pi y$. The fund first uses the liquid asset holdings before resorting to risky asset sales. I assume that investing all units of the liquid asset is insufficient to drive the marginal return of the short-term project f down to 1. Therefore, the fund sets $l = m_f$ and in addition chooses to sell $q \geq 0$ units of the liquid asset. Then, q is pinned down by the first-order condition

$$f'\left(qp + m_f\right) = \frac{\pi y}{p}$$

If the price of the risky asset is sufficiently high, we may have a corner solution with $q = 1 - m_f$. This is the case if the price p satisfies $f'\left(\left(1 - m_f\right)p + m_f\right)p \geq \pi y$.

Case 2: $p = \pi y$. Since the opportunity cost of the two financing modes is equal, I assume that the fund first invests all of its liquid asset holdings before resorting to risky asset sales without loss of generality. Again, I assume that the liquid asset is insufficient to drive the marginal return of the short-term project down to 1 so that the fund additionally sells some of the risky asset. The representative fund's supply is pinned down by the first-order condition with respect to q with $p = \pi y$ and $l = m_f$:

$$f'\left(q\pi y + m_f\right) = 1.$$

However, if the fundamental price of the risky asset is sufficiently high such that equation (2) with $p = \pi y$ holds, investors' problem has a corner solution with $q = 1 - m_f$.

This shows that focusing on the case with $l = m_f$ can be easily rationalized in the context of this model.

Asset market equilibrium. Region A: Assuming that the equilibrium is located on the increasing part of the supply curve and the downward-sloping part of the demand curve implies $p < \pi y, q^D = \frac{m_l}{p}, pq = M_l$. Supply is determined by investors' FOC such that I obtain

$$q^* = \frac{M_l f'\left(M_l + M_f\right)}{\pi y}, p^* = \frac{\pi y}{f'\left(M_l + M_f\right)}.$$

Consistency of this result with the initial assumption requires $q^* \leq 1 - M_f$, which holds if $\frac{\pi y}{f'\left(M_l + M_f\right)} \geq \frac{M_l}{\left[1 - M_f\right]}$, and $p^* < \pi y$ which reduces to $1 \leq f'\left(M_l + M_f\right)$.

Region B: Assuming that the equilibrium is located on the flat part of the demand curve and the upwards-sloping supply curve implies that mutual funds' optimal asset supply condition with $p = \pi y$ pins down the equilibrium:

$$q^* = \frac{\left(f'\right)^{-1}(1) - M_f}{\pi y}, p^* = \pi y.$$

Consistency with the initial assumptions requires $q^* < 1 - M_f$, which reduces to $f'\left(\pi y\left[1-M_f\right]+M_f\right) < 1$, and $pq < M_l$, which yields $1 > f'\left(M_l + M_f\right)$.

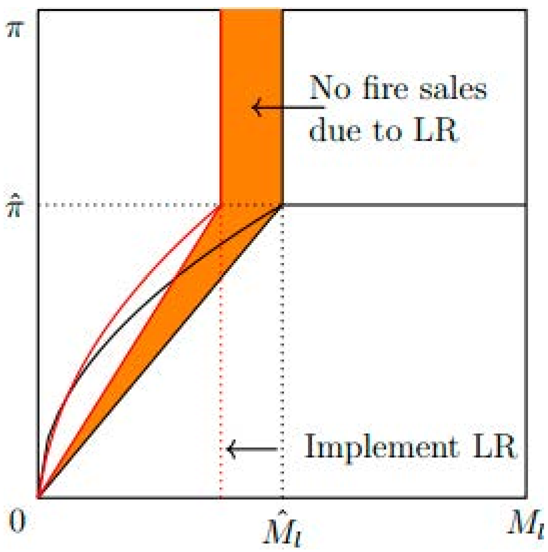
Region C: Assuming that the equilibrium is located on the flat part of the demand curve and the vertical supply curve implies $p^* = \pi y$ and $q^* = 1 - M_f$. This is consistent with the initial assumptions if $f'\left(\pi y\left[1-M_f\right]+M_f\right) \geq 1$ and $\pi y < \frac{M_l}{\left(1-M_f\right)}$ hold.

Region D: Assuming that the equilibrium is located on the downward-sloping part of the demand curve and the vertical supply curve implies $q^* = 1 - M_f, p^* = \frac{M_l}{1-M_f}$. At this price, investors indeed sell all of their asset if $\frac{\pi y}{f'\left(M_l + M_f\right)} \leq \frac{M_l}{1-M_f}$, and the price is below the asset's fundamental value if $\pi y \geq \frac{M_l}{1-M_f}$.

The preceding analysis suggests two thresholds which help to characterize the equilibrium regions in the asset market. As opposed to the model without liquidity regulation, the thresholds are now functions of the exogenous liquidity holdings of the mutual fund. First, a threshold on market liquidity $\widehat{M_l}$, implicitly defined by the equation $f'\left(\widehat{M_l} + M_f\right) = 1$, such that the risky asset trades at its fundamental price regardless of the realized asset quality if $M_l \geq \widehat{M_l}$. Second, a threshold on asset quality $\hat{\pi}$ implicitly defined by the equation $f'\left(\hat{\pi} y\left[1-M_f\right]+M_f\right) = 1$, such that investors keep some of their risky asset holdings regardless of market liquidity if $\pi > \hat{\pi}$. In the parametric example with $f'(k) = 1/\sqrt{k}$, we have $\widehat{M_l} = 1 - M_f$ and $\hat{\pi} = 1/y$, that is, the liquidity requirement has no impact on the latter threshold.

Figure B.1 illustrates the equilibrium characterization on the asset market. It shows that introducing liquidity regulation $\left(M_f > 0\right)$ changes the boundaries of the equilibrium regions and decreases the likelihood of fire sales for a given level of market liquidity.

Equilibrium characterization under liquidity regulation FIGURE B.1



Note: This figure illustrates how liquidity regulation (LR) changes the boundaries of the equilibrium regions in the asset market. The shaded area highlights states of the economy in which the introduction of LR ($M_f > 0$) prevents fire sales.

Marginal private value of liquidity. Recall that liquidity providers' profit from purchasing the risky asset at date 1 is

$$v^p = q^D\left(p^*;\pi,m_l\right)\left[\pi y - p^*\right]$$

and the marginal private value of liquidity at date 1 is $v^{p'} = \partial v^p / \left(\partial m_l\right)$. I now derive $v^{p'}$ in each of the four equilibrium regions. First, Regions B and C feature fundamental pricing with $p^* = \pi y$ which trivially implies $v^p = v^{p'} = 0$. Instead, Region A features fire sales with $p^* = \dfrac{\pi y}{f\left(M_l + M_f\right)}$, implying $q^D\left(p^*;\pi,m_l\right) = m_l / p^*$ and

$$v^p = m_l\left[f\left(M_l + M_f\right) - 1\right] \quad \text{and} \quad v^{p'} = f\left(M_l + M_f\right) - 1.$$

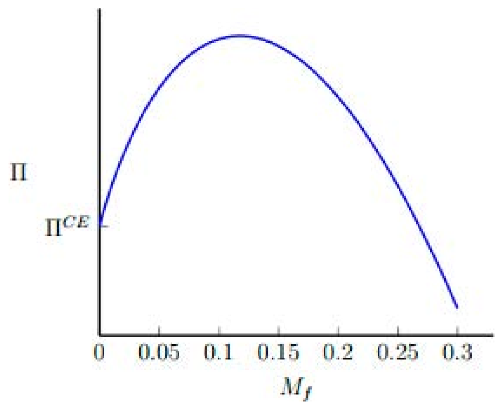
Region D features fire sales with $p^* = \dfrac{M_l}{1 - M_f}$, which implies $q^D\left(p^*;\pi,m_l\right) = m_l / p^*$ and

$$v^p = m_l\left[\frac{\pi y\left(1 - M_f\right)}{M_l} - 1\right] \quad \text{and} \quad v^{p'} = \frac{\pi y\left(1 - M_f\right)}{M_l} - 1.$$

Note that when fire sales occur, I obtain $\dfrac{\partial v^{p'}}{\partial M_f} =< 0$, that is, liquidity providers' marginal value of liquidity is decreasing in the liquidity holdings of the mutual fund.

Social welfare. Figure B.2 depicts the impact of the liquidity buffer on social welfare. Starting from the unregulated equilibrium with $M_f = 0$ and $\Pi = \Pi^{CE}$, increasing the liquidity buffer initially improves welfare. This is due to the buffer's positive effect on market liquidity and the decreased likelihood of fire sales. However, there is a threshold on the liquidity buffer after which increasing the buffer decreases social welfare, potentially even below the unregulated competitive benchmark. This is because the liquidity buffer forces a reduction in the investment in the risky asset, which is a positive NPV investment. The trade-off between the buffers' positive effect on market liquidity and the negative effect on the positive NPV investment leads to the depicted hump-shaped relationship between social welfare and the liquidity buffer.

Social welfare under liquidity regulation FIGURE B.2



Note: This figure illustrates the effect of the liquidity buffer on social welfare Π. The unregulated competitive equilibrium obtains for $M_f = 0$ and features $\Pi = \Pi^{CE}$.

Model with endogenous mutual fund liquidity

This section derives the equilibrium of the model in which mutual funds can invest their collected resources at date 0 in a portfolio consisting of m_f units of the liquid asset and $1-m_f$ units of the risky asset. Subsequently, it presents a constrained efficient benchmark to analyze the efficiency of competitive mutual funds' liquidity holdings.

Competitive equilibrium

The analysis of the possible configurations of equilibrium on the asset market for given liquidity levels and asset quality coincides with the analysis under an exogenous liquidity requirement. Therefore, liquidity providers' problem at date 0 remains unchanged, and I can focus directly on mutual funds' portfolio choice at date 0.

Recall the definition of mutual funds' (expected) payoff from holding the liquid asset at the start of date 1 conditional on the realized π:

$$\rho^p\left(\pi;m_f\right) \equiv f\left(q^S\left(p^*;\pi,m_f\right)p^* + m_f\right) - q^S\left(p^*;\pi,m_f\right)\pi y.$$

I begin by calculating $\rho^p\left(\pi;m_f\right)$ and $\rho^{p'}\left(\pi;m_f\right) \equiv \partial\rho^p\left(\pi;m_f\right)/\partial m_f$ in each of the four candidate equilibria on the asset market.

<u>Region A</u>: $p^* = \dfrac{\pi y}{f'\left(M_l+M_f\right)}$ and $q^S\left(p^*;\pi,m_f\right) = \dfrac{f'\left(M_l+M_f\right)\left[M_l+M_f-m_f\right]}{\pi y}$ yield

$$\rho^p = f'\left(M_l+M_f\right)\left[1-M_l-M_f+m_f\right] \quad \text{and} \quad \rho^{p'} = f'\left(M_l+M_f\right).$$

<u>Region B</u>: $p^* = \pi y$ and $q^S\left(p^*;\pi,m_f\right) = f'\left(M_l+M_f\right)\left[M_l+M_f-m_f\right]/\left(\pi y\right)$ yield

$$\rho^p = f\left(\left(f'\right)^{-1}(1)\right) - \left(f'\right)^{-1}(1) + m_f \quad \text{and} \quad \rho^{p'} = 1.$$

<u>Region C</u>: $p^* = \pi y$ and $q^S\left(p^*;\pi,m_f\right) = 1-m_f$ yield

$$\rho^p = f\left(\left(1-m_f\right)\pi y + m_f\right) - \left(1-m_f\right)\pi y \quad \text{and}$$

$$\rho^{p'} = \left[1-\pi y\right]f'\left(\left(1-m_f\right)\pi y + m_f\right) + \pi y.$$

<u>Region D</u>: $p^* = \dfrac{M_l}{1-M_f}$ and $q^S\left(p^*;\pi,m_f\right) = 1-m_f$ yield

$$\rho^p = f\left(\frac{\left(1-m_f\right)M_l}{1-M_f} + m_f\right) - \left(1-m_f\right)\pi y \quad \text{and}$$

$$\rho^{p'} = \left[1-\frac{M_l}{1-M_f}\right]f'\left(\frac{\left(1-m_f\right)M_l}{1-M_f} + m_f\right) + \pi y.$$

The expected marginal private value of mutual fund liquidity at date 0 is then determined by forming expectations about the marginal private value in each of the possible configurations of equilibrium.

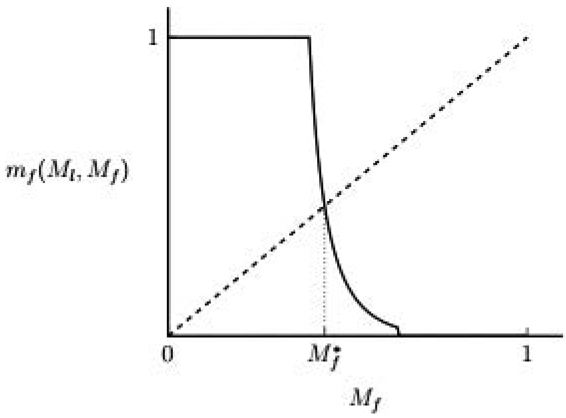
The investment in the liquid asset by mutual funds at date 0 in the competitive equilibrium must satisfy the first-order condition

$$\bar{\pi} y = \rho_0^p(M),$$

which implicitly defines a value of m_f^* for any $\left(M_f, M_l\right)$, say $m_f\left(M_f, M_l\right)$. In equilibrium, the aggregate level of mutual fund liquidity M_f^* is the unique fixed point of $m_f\left(M_f^*, M_l^*\right) = M_f^*$, where M_l is the similarly derived solution to liquidity providers' fixed-point problem at date 0 as described in the previous section.

Figure C.3 shows that mutual funds' individual choice of liquidity m_f is a (weakly) declining function of aggregate liquidity M_f. The intuition follows the same logic as in the case of liquidity providers' fixed-point problem: liquidity is especially valuable when market liquidity $M_f + M_l$ is low such that fire sales at date 1 are likely. In this situation, holding more of the liquid asset allows accommodating investor redemptions without resorting to costly risky asset fire sales. As market liquidity increases and fire sales become less likely, the benefits of holding liquidity decline, and the implicit cost in the form of foregone returns from investing in the risky asset dominates.

Fixed point in mutual fund liquidity provision FIGURE C.3



Note: This figure illustrates how mutual funds' optimal choice of liquidity m_f at date 0 varies with aggregate liquidity in the hands of mutual funds M_f, assuming liquidity providers hold some intermediate level of M_l. The dashed diagonal line depicts the 45-degree line. The equilibrium level of mutual fund liquidity is characterized by the intersection of the 45-degree line and $m_f(M_f, M_l)$.

Second best with endogenous mutual fund liquidity

This section derives the solution to the constrained planner's problem presented in Section 5.2. Recall that the interior solution to the constrained planner's problem is characterized by the two first-order conditions

$$g'(w - M_l) = \quad v_0^s(\pi;\bar{M}) \equiv \mathbb{E}\left[\frac{\partial v^s(\pi;\bar{M})}{\partial M_l}\right]$$
$$\bar{\pi} y = \quad \rho_0^s(\pi;\bar{M}) \equiv \mathbb{E}\left[\frac{\partial \rho^s(\pi;\bar{M})}{\partial M_f}\right].$$

First, I derive the marginal social value of liquidity for mutual funds and liquidity providers in the four possible equilibrium configurations at date 1.

Regions A and D: Fire sales of the risky asset and $p^* q^* = M_l$ yield

$$v^s(\pi;\bar{M}) = f(M_l + M_f) \quad \text{and} \quad v^{s'}(\pi;\bar{M}) = f'(M_l + M_f)$$
$$\rho^s(\pi;\bar{M}) = f(M_l + M_f) + M_l \quad \text{and} \quad \rho^{s'}(\pi;\bar{M}) = f'(M_l + M_f).$$

Region B: $p^* = \pi y$ and $q^* = \frac{(f')^{-1}(1) - M_f}{\pi y}$ yield $p^* q^* = (f')^{-1}(1) - M_f$ and

$$v^s(\pi;\bar{M}) = M_l + f\left((f')^{-1}(1)\right) - (f')^{-1}(1) + M_f \quad \text{and} \quad v^{s'}(\pi;\bar{M}) = 1$$
$$\rho^s(\pi;\bar{M}) = f\left((f')^{-1}(1)\right) + (f')^{-1}(1) + M_f \quad \text{and} \quad \rho^{s'}(\pi;\bar{M}) = 1.$$

Region C: $p^* = \pi y$ and $q^* = 1 - M_f$ yield $p^* q^* = \pi y(1 - M_f)$ and

$$v^s(\pi;\bar{M}) = M_l + f\left(\pi y + (1 - \pi y) M_f\right) - \pi y(1 - M_f) \quad \text{and} \quad v^{s'}(\pi;\bar{M}) = 1$$
$$\rho^s(\pi;\bar{M}) = f\left(\pi y(1 - M_f) + M_f\right) + \pi y(1 - M_f) \quad \text{and} \quad \rho^{s'}(\pi;\bar{M}) = (1 - \pi y)$$
$$f'\left(\pi y + (1 - \pi y) M_f\right) + \pi y.$$

The expected marginal social value of liquidity for mutual funds at date 0 is then determined by forming expectations about the marginal private value in each of the possible configurations of equilibrium.

Equilibrium with redemption gates

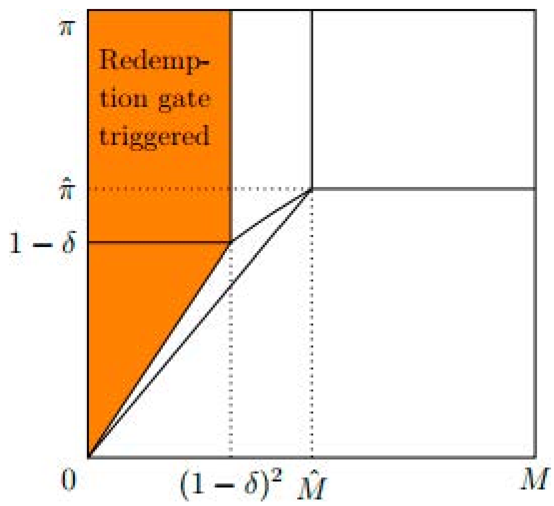
This section derives the equilibrium under redemption gates. As the policy only affects equilibrium regions in which fire sales occur, I focus on highlighting how the regulated equilibrium differs from the competitive benchmark in equilibrium regions A and D.

Region A: In the unregulated benchmark, this candidate equilibrium features $p^* = \frac{\pi y}{f'(M)}$ and $q^* = \frac{M f'(M)}{\pi y}$. It obtains if $\pi y \geq M f'(M)$ and $1 \leq f'(M)$ hold. Additionally, it must not trigger the activation of redemption gates: $\frac{1}{f'(M)} \geq 1 - \delta$. If the latter condition is violated, the regulatory constraint binds and implies $p^* = (1 - \delta)\pi y$ and $q^* = \frac{M}{(1 - \delta)\pi y}$. This equilibrium is consistent with the initial assumptions of Region A if $q^* \leq 1$, which reduces to $M \leq (1 - \delta)\pi y$.

Region D: In the absence of regulatory interventions, this candidate equilibrium features $p^* = M$ and $q^* = 1$ and obtains if $\pi y \le Mf'(M)$ and $\pi y \ge M$ hold. At this price, redemption gates are not triggered if $M > (1-\delta)\pi y$. Instead, if this constraint is violated, the regulatory constraint binds, and I obtain $p^* = (1-\delta)\pi y$ and $q^* = 1$. At this equilibrium price, mutual funds' indeed choose to sell all of their risky asset holdings if $f'((1-\delta)\pi y) \ge \frac{1}{(1-\delta)}$. Moreover, the risky asset price is indeed below fundamentals if $M \le (1-\delta)\pi y$.

Figure D.4 characterizes the equilibrium on the asset market under redemption gates. It illustrates that redemption gates are triggered in especially severe fire-sale episodes, that is, when market liquidity tends to be very low.

Equilibrium characterization under redemption gates FIGURE D.4



Note: This figure illustrates how a redemption gate changes the boundaries of the equilibrium regions in the asset market. The shaded area highlights states of the economy in which fire sales trigger the activation of the gate.

I now compute the marginal private value of liquidity under redemption gates. Recall that liquidity providers' profit from purchasing the risky asset at date 1 is

$$v^p = q^D(p^*;\pi,m)\left[\pi y - p^*\right].$$

and the marginal private value of liquidity at date 1 is $v^{p'} = \frac{\partial v^p}{\partial m}$. First, Regions B and C feature fundamental pricing which trivially implies $v^p = v^{p'} = 0$. As long as redemption gates are not triggered, Region A features fire sales with $p^* = \frac{\pi y}{f'(M)}$ and $q^D(p^*;\pi,m) = m / p^*$, which yields

$$v^p = m_l\left[f'(M) - 1\right] \quad \text{and} \quad v^{p'} = f'(M) - 1.$$

If the gate is activated, I obtain $p^* = (1-\delta)\pi y$, $q^D(p^*;\pi,m) = m / p^*$, and

$$v^p = \frac{m\delta}{1-\delta} \quad \text{and} \quad v^{p'} = \frac{\delta}{1-\delta}.$$

Region D features fire sales with $p^* = M$ implying $q^D(p^*;\pi,m) = m / p^*$ in the absence of a binding regulatory constraint such that

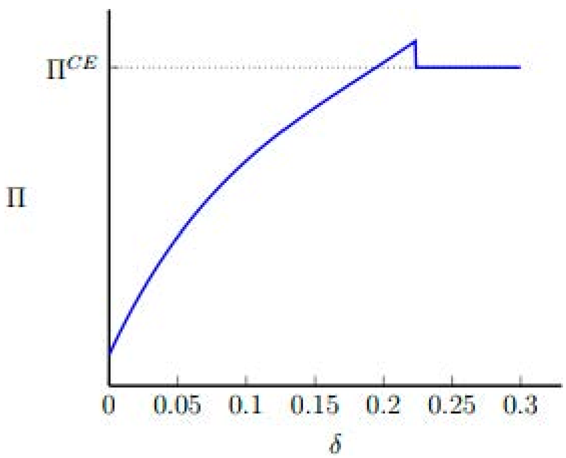
$$v^p = m\left[\frac{\pi y}{M} - 1\right] \quad \text{and} \quad v^{p'} = \frac{\pi y}{M} - 1.$$

If the redemption gate is triggered, I obtain $p^* = (1-\delta)\pi y$, $q = 1$, and

$$v^p = \frac{m\delta}{1-\delta} \quad \text{and} \quad v^{p'} = \frac{\delta}{1-\delta}.$$

Figure D.5 shows the effect of redemption gates on social welfare. If redemption gates are never triggered in equilibrium (high δ), welfare in the regulated equilibrium coincides with the unregulated benchmark $\Pi = \Pi^{CE}$. Conversely, if the threshold for triggering redemption gates is low, only mild fire sales occur in equilibrium, and welfare is well below the competitive benchmark due to the decline in market liquidity. If the policy is calibrated such that the resulting market liquidity is only marginally below the competitive benchmark, redemption gates increase social welfare by mitigating the adverse impact of fire sales on investors. However, calibrating the threshold δ correctly appears to be a challenge in practice and carries the risk of decreasing welfare.

Welfare under redemption gates FIGURE D.5



Note: This figure illustrates how social welfare varies with the threshold on market distress δ governing the activation of redemption gates. Π^{CE} denotes the unregulated benchmark.

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